University of Kentucky  
Department of Electrical and Computer Engineering  

EE421G: Signals and Systems I – Fall 2007  

Problem Set 12  
Issued: November 30, 2007    Due: December 5, 2007 (In class)  

**Reading Assignments:**  
Read Chapter 9 of Chen  

**Paper and Pencil Assignments:**  
1. Problem 9.1  
Solution:  
\[ H(z) = Z[h[n]] = \frac{z}{z-1} + 2\frac{z}{z-0.8} - 3\frac{z}{z+0.7} \]  
\[ = \frac{z(4.7z-4.36)}{(z-1)(z-0.8)(z+0.7)} = \frac{4.7z^2 - 4.36z}{z^3 - 1.1z^2 - 0.46z + 0.56} \]  

2. Problem 9.2  
Solution:  
(a)  
\[ H(z) = \frac{Y(z)}{U(z)} = \frac{z^2 - z - 2}{2z^2 + 4z + 10} \]  
(b)  
\[ H(z) = \frac{Y(z)}{U(z)} = \frac{z^2 + 3z + 2}{2z^3 + 4z^2 + 10z} \]  
(c)  
\[ H(z) = \frac{Y(z)}{U(z)} = \frac{z^3 + 1}{z^3 + 1} \]  

They equal these in Prob. 6.2 if \( z \) is replaced by \( s \). They are all proper. (a) is biproper; (b) and (c) are strictly proper.  

3. Problem 9.3  
Solution:  
\[ z^4V(z) + 3z^3V(z) + 10V(z) = 2z^2R(z) + 5zR(z) + 3R(z) \]  
\[ v[n+4] + 3v[n+3] + 10v[n] = 2r[n+2] + 5r[n+1] + 3r[n] \]  

4. Problem 9.6  
Solution:
Impulse response = $Z^{-1} [H(z)]$

$H(z) = \frac{0.9}{(z+1)(z-0.8)} - \frac{0.5}{z+1} + \frac{0.5}{z-0.8}$

$H(z) = -0.5 \frac{z}{z+1} + 0.5 \frac{z}{z-0.8}$

$h[n] = -0.5(-1)^n + 0.5(0.8)^n, n \geq 0$

Step response $Y(z) = H(z) \cdot \frac{z}{z-1} = \frac{0.9z^2}{(z+1)(z-0.8)(z-1)}$

$Y(z) = -0.25 + \frac{-2}{z+1} + \frac{2.25}{z-0.8}$

$Y(z) = -0.25z + \frac{-2z}{z-0.8} + \frac{2.25z}{z-1}$

$y[n] = -0.25(-1)^n - 2(0.8)^n + 2.25(1)^n, n \geq 0$

5. Problem 9.7
Solution:

$H(z) = \frac{z+1}{-8z+10}, u[n] = \sin 0.1n$

$U(z) = \frac{(\sin 0.1)z}{z^2 - 2(\cos 0.1)z + 1} = \frac{0.0998z}{z^2 - 1.99z + 1} = \frac{0.0998z}{(z-e^{j0.1})(z-e^{-j0.1})}$

$Y(z) = -0.3742 + \frac{0.4555e^{-j1.1475}}{z-1.25} + \frac{0.4555e^{j1.1475}}{z-e^{j0.1}}$

$Y(z) = H(z)U(z) = \frac{0.0998z(z+1)}{-8(z-1.25)(z-e^{j0.1})(z-e^{-j0.1})}$

$y[n] = -0.3742(1.25)^n + 0.4555e^{-j1.1475}(e^{j0.1})^n + 0.4555e^{j1.1475}(e^{-j0.1})^n$

$Y(1.1475 \approx 1.15)$

$y[n] = -0.374(1.25)^n + 0.91\cos(0.1n - 1.15)$

$= -0.374(1.25)^n + 0.91 \sin(0.1n - 1.15 + \pi/2)$

$= -0.374(1.25)^n + 0.91 \sin(0.1n + 0.42)$

for $n \geq 0$

6. Problem 9.10
Solution:

7. Problem 9.12
Solution:
To show $|H_1(e^{j\omega})| = |H_2(e^{j\omega})|$, we show

$|10z - 8| = |8 - 10z|$, for $z = e^{j\omega}$

$|10z - 8|_{z = e^{j\omega}} = |z(10 - 8z^{-1})|_{z = e^{j\omega}}$

$= |e^{j\omega}(10 - 8e^{-j\omega})| = |10 - 8e^{-j\omega}|$

$= |10 - 8(\cos\omega - j\sin\omega)| = |10 - 8\cos\omega + j8\sin\omega|$

$= \left[(10 - 8\cos\omega)^2 + (8\sin\omega)^2\right]^{1/2}$

$|\frac{-8z + 10}{1 - e^{j\omega}}|$

$= |e^{j\omega}(8 + j8\sin\omega)|$

$= |8(\cos\omega + j\sin\omega) + 10| = |10 - 8\cos\omega - j8\sin\omega|$

$= \left[(10 - 8\cos\omega)^2 + (-8\sin\omega)^2\right]^{1/2}$

Thus $H_1(z)$ and $H_2(z)$ have the same magnitude response. Their phase responses however are different.