University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 1

Issued: August 22, 2007 Due: August 31, 2008 (In class)

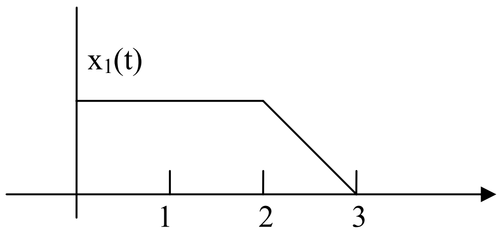
Reading Assignments:

Read Chapter 1 of Chen

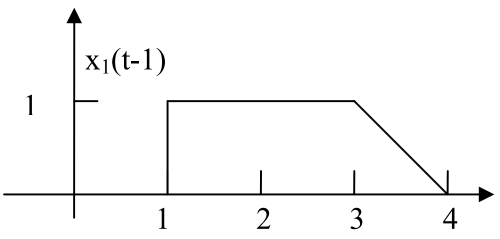
Paper and Pencil Assignments:

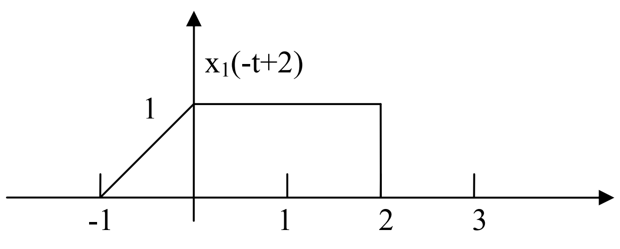
Please do the following problems at the end of chapter 1:

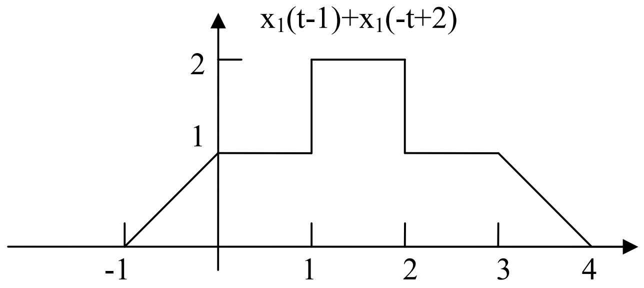
1) Problem 1.3: Consider the signal $x_1(t)$ shown in the following figure. Plot $x_1(t-1)$, $x_1(-t+2)$, $x_1(t-1)+x_1(-t+2)$, $x_1(t-1)-x_1(-t+2)$, and $x_1(t-1)x_1(-t+2)$.

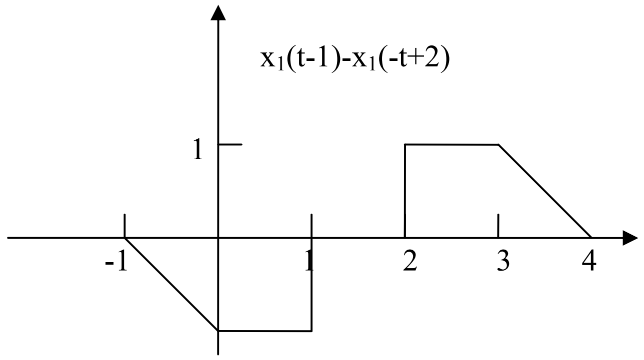


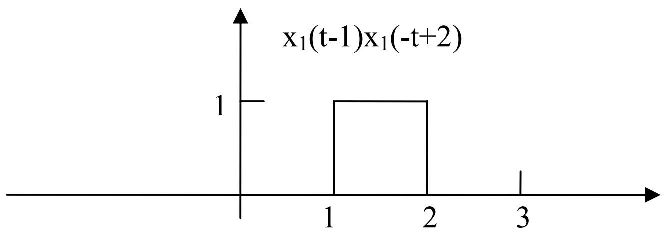
Solution:



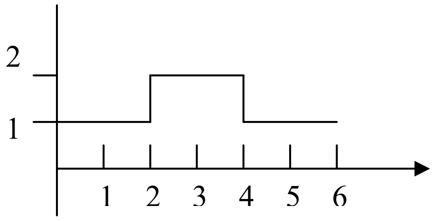




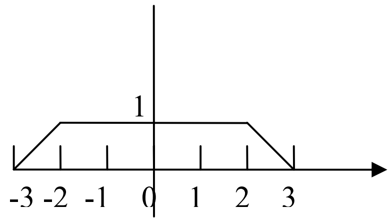




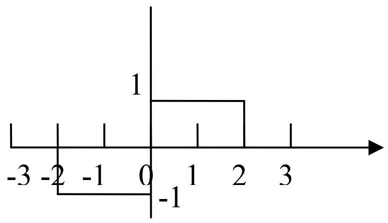
2) Problem 1.6: Express the signals in the following figures in terms of step and ramp functions.



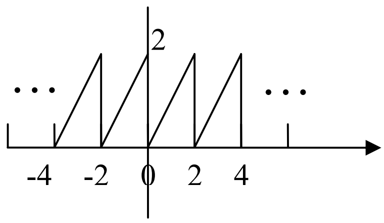
(a)



(b)



(c)

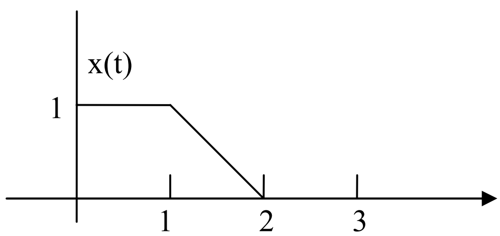


(d)

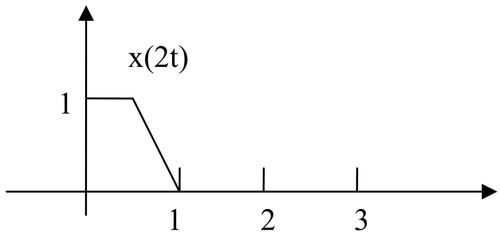
Solution:
(a) q(t)+q(t-2)-q(t-4)
(b) [r(t+3)-r(t+2)]-[r(t-2)-r(t-3)]
(c) –q(t+2)+2q(t)-q(t-2) or q(t)-q(t-2)-[q(-t)-q(-t-2)]
(d) The triangle in [0,2] is x(t)=r(t)-r(t-2)-2q(t-2);

$$\sum_{k=-\infty}^{\infty} x(t-2k) = \sum_{k=-\infty}^{\infty} r(t-2k) - r\left(t-2k-2\right) - 2q\left(t-2k-2\right)$$

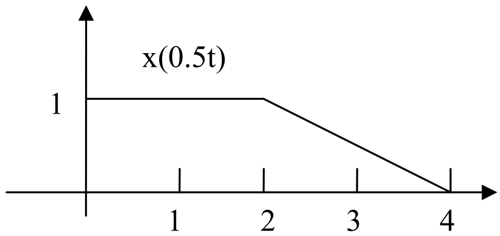
3) Problem 1.7: Consider the signal in the following figure. It starts from t=0 and ends at t=2 and is said to have time duration 2.
   a. Plot x(2t). What is its time duration?
   b. Plot x(0.5t). What is its time duration?
   c. Show that if a>1, then the time duration of x(at) is smaller than that of x(t). This speeds up the signal and is call time compression.
   d. Show that if 0<a<1, then the time duration of x(at) is larger than that of x(t). This slows down the signal and is called time expansion.



Solution:
(a) Its time duration is 1.



(b) Its time duration is 4.



(c) x(at) with a>1 speeds up the signal or achieves time compression.
(d) x(at) with 0<a<1 slows down the signal or achieves time expansion.

4) Problem 1.9: Consider a signal x(t). Define

$$x_e(t) = \frac{x(t)+x(-t)}{2} \text{ and } x_o(t) = \frac{x(t)-x(-t)}{2}$$

Show that $x_e(t)$ is even and $x_o(t)$ is odd. Note: a signal x(t) is even if x(t)=x(-t) and is odd if x(t)=-x(-t).

Solution:

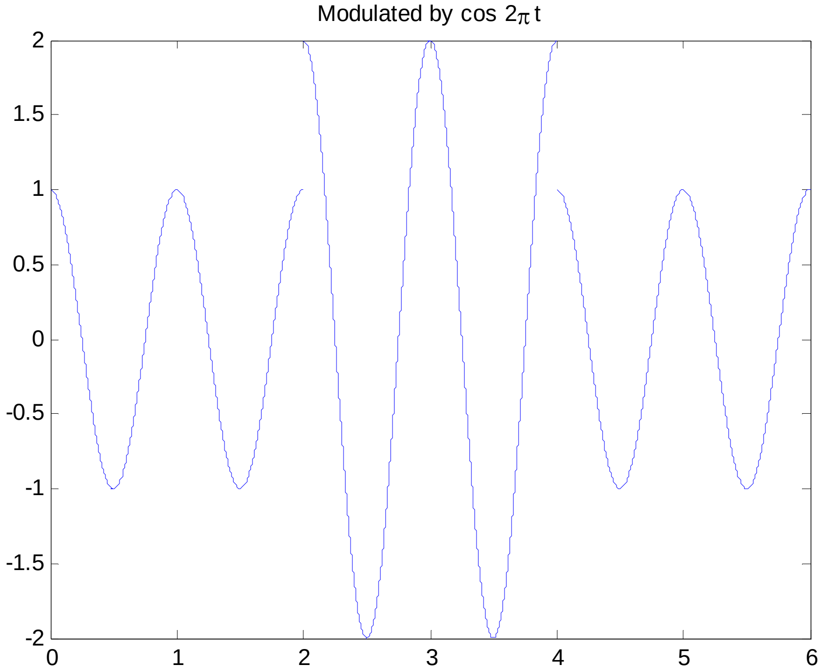
$$x_e(-t) = \frac{x(-t)+x(-(-t))}{2} = \frac{x(-t)+x(t)}{2} = x_e(t)$$, so $x_e(t)$ is even;

$$x_0(-t) = \frac{x(-t)-x(-(-t))}{2} = -\frac{x(t)-x(-t)}{2} = -x_0(t)$$, so $x_o(t)$ is odd.

5) Problem 1.13: Sketch the signal in Figure a in Problem 2 modulated by cos2πt.

Solution:

The period of cos2πt is 1.



6) Problem 1.14: Compute

a. $\int_0^9 [\cos \pi\tau]\delta(\tau-3)d\tau$

b. $\int_5^9 [\cos \pi\tau]\delta(\tau-3)d\tau$

c. $\int_{-\infty}^{\infty} [\cos(t-\tau)]\delta(\tau+3)d\tau$

d. $\int_0^{\infty} [\cos(t-\tau)]\delta(\tau+3)d\tau$

e. $\int_{-\infty}^{0} [\cos(t-\tau)]\delta(\tau+3)d\tau$

Solution:

a. $\int_0^9 [\cos \pi\tau]\delta(\tau-3)d\tau = \cos 3\pi = -1$

b. $\int_5^9 [\cos \pi\tau]\delta(\tau-3)d\tau = 0$

c. $\int_{-\infty}^{\infty} [\cos(t-\tau)]\delta(\tau+3)d\tau = \cos(t+3)$

d. $\int_0^{\infty} [\cos(t-\tau)]\delta(\tau+3)d\tau = 0$

e. $\int_{-\infty}^{0} [\cos(t-\tau)]\delta(\tau+3)d\tau = \cos(t+3)$

7) Problem 1.20: Show

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

where n_0 is a fixed integer, and

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

They are the DT counterparts of the sifting property.

Solution:

Because $\delta[n-n_0]$ is 0 for all $n \neq n_0$, we have $x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

Using the above, we have

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = \sum_{n=-\infty}^{\infty} x[n_0]\delta[n-n_0] = x[n_0]\sum_{n=-\infty}^{\infty} \delta[n-n_0] = x[n_0]\cdot 1 = x[n_0]$$

8) ~~Problem 1.22: Consider the CT signal~~

$$x(t) = 2 + \sin 1.4t - 4\cos 2.1t$$

~~Is it periodic? If yes, find its fundamental period. (a) Express it using exclusively sine functions. What are their frequencies, corresponding magnitudes, and phases? (b) Express it using exclusively cosine functions. What are their frequencies, corresponding magnitudes, and phases?~~

9) ~~Problem 1.31: Consider~~ $\cos \omega_k n$ ~~with T=1 and~~

$$\omega_k = \frac{2\pi k}{N}$$

~~where N is a given positive integer and k is an integer ranging from -∞ to ∞. How many different~~ $\cos \omega_k n$ ~~are there? What are their frequencies?~~

10) ~~Problem 1.34: Consider the CT signal~~ $x(t) = \sin 50t + 2\sin 70t$ ~~. Verify that its sampled sequence x(nT) is~~ $-\sin(50nT)$ ~~if T is selected as T=π/60. Can you determine the two frequencies of x(t) from x(nT)? What is the sampled sequence of x(t) if T=π/45? Can you determine the two frequencies of x(t) from this sampled sequence? Repeat the question for T=π/180.~~