University of Kentucky
Department of Electrical and Computer Engineering

# EE421G: Signals and Systems I – Fall 2007

Problem Set 2

Issued: September 2, 2007 Due: September 10, 2008 (In class)

**Reading Assignments:**

Read Chapter 1.7-1.9 and 2.1-2.3 of Chen

**Computer Assignments:**

1) Try out different demos on the webpage "*Listen to Fourier Series*" at http://www.jhu.edu/~signals/listen-new/listen-newindex.htm and answer the following questions:
   a. "Introduction": Compute the logarithmic difference of successive notes. Take the base-10 logarithm of each frequency before computing their differences.

Solution:

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MATLAB code:
Note=[220, 233, 247, 262, 277, 294, 311, 330, 349, 370, 392, 415];
freq_log=log10(Note)

for i=1:(length(freq_log)-1)
    diff(i)=freq_log(i+1)-freq_log(i);
end
diff

Result:
freq_log =
2.3424    2.3674    2.3927    2.4183    2.4425    2.4683    2.4928
2.5185    2.5428    2.5682    2.5933    2.6180
diff =
0.0249    0.0253    0.0256    0.0242    0.0259    0.0244    0.0258
0.0243    0.0254    0.0251    0.0248

So the logarithmic difference of successive notes is around 0.025.
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   b. "Additional Tones": This section shows the amplitude responses of playing the note F using different instruments. What do you think the amplitude response for each instrument would be if the note G is played?

Solution:

The amplitude response of note G for each instrument would be the same as that of note F.

c. "Harmonic Contribution": Is there a difference in pitch between the low-passed signal and the high-passed signal? Explain such difference.

Solution:
Yes, there is a difference in pitch between the low-passed signal and the high-passed signal. The pitch is dependent primarily on the frequency of the sound waves. High-passed signal has the high frequency components, so the pitch is higher. Low-passed signal has the low frequency components, so the pitch is lower.

2) Read "*Discrete-Time Frequency*" at http://www.jhu.edu/~signals/dtphasor/index.htm and use the applet to answer problem 4 below for the cases when N = 4, 8 and 16.

If the Java applets in the above webpages are not running properly, install the latest Java Runtime Environment (JRE) from http://java.sun.com/javase/downloads/index.jsp

**Paper and Pencil Assignments:**

1) Problem 1.22: Consider the CT signal

$$x(t) = 2 + \sin 1.4t - 4\cos 2.1t$$

Is it periodic? If yes, find its fundamental period. (a) Express it using exclusively sine functions. What are their frequencies, corresponding magnitudes, and phases? (b) Express it using exclusively cosine functions. What are their frequencies, corresponding magnitudes, and phases?

Solution:
Yes. Its fundamental frequency is 0.7 rad/s, and its fundamental period is $2\pi/0.7$ =8.976.
Using 2= $2\sin(0\cdot t+\pi/2)=2\cos(0\cdot t)$

$\pm\cos(\omega t)=\sin(\omega t\pm\pi/2)$

$\pm\sin(\omega t)=\cos(\omega t\mp\pi/2)$

we have

(a) $x(t)=2\sin(0\cdot t+\pi/2)+\sin(1.4t)+4\sin(2.1t-\pi/2)$

| | | | |
|---|---|---|---|
| Frequency | 0 | 1.4 | 2.1 |
| Magnitude | 2 | 1 | 4 |
| Phase | $\pi/2$ | 0 | $-\pi/2$ |

(b) $x(t)=2\cos(0\cdot t)+\cos(1.4t-\pi/2)+4\cos(2.1t+\pi)$

| | | | |
|---|---|---|---|
| Frequency | 0 | 1.4 | 2.1 |
| Magnitude | 2 | 1 | 4 |

| | | | |
|---|---|---|---|
| Phase | 0 | $-\pi/2$ | $\pi$ |

2) Problem 1.24: Is the signal

$$x(t) = 2 + \sin 2t - 3\cos \pi t$$

periodic? Can it be expressed using complex exponentials?

Solution:
The function is not periodic because 2 and $\pi$ have no common divisor. It can be expressed using complex exponentials as
$x(t) = 2 + 0.5e^{-j\pi/2}e^{j2t} + 0.5e^{j\pi/2}e^{-j2t} + 1.5e^{j\pi}e^{j\pi t} + 1.5e^{j\pi}e^{-j\pi t}$

3) Problem 1.27: Are the sequences $\sin(6.9\pi n)$, $\sin(4.9\pi n)$, $\sin(2.9\pi n)$, $\sin(0.9\pi n)$, $\sin(-1.1\pi n)$ and $\sin(-3.1\pi n)$, for T=1 the same (DT) sequence? If not, how many different sequences are there? Find their ~~frequencies~~. Periods.

Solution:
For T=1, the Nyquist frequency range is ($-\pi$, $\pi$].
$6.9\pi = 4.9\pi = 2.9\pi = 0.9\pi = -1.1\pi = -3.1\pi \left(\text{mod } 2\pi/T = 2\pi\right)$
Thus they all denote the same (DT) sequence.
For n=0:5, any of these sequences will yield
0 0.309 -0.5878 0.809 -0.9511 1

Because $0.9\pi$ lies inside Nyquist frequency range, the
$\sin(0.9\pi n) = \sin(0.9\pi nT)$ with T=1
has frequency $0.9\pi$ rad/s, so the period is $2\pi/0.9\pi = 20/9$=2.22.

~~4) Problem 1.31: Consider $\cos\omega_k n$ with T=1 and~~

$$\omega_k = \frac{2\pi k}{N}$$

~~where N is a given positive integer and k is an integer ranging from -∞ to ∞. How many different $\cos\omega_k n$ are there (in terms of N)? What are their frequencies (in terms of N)?~~

~~5) Problem 1.35: What are the sample sequences of $x(t) = \cos 50t + 2\sin 70t$ for $T = \pi/45$, $\pi/30$ and $\pi/180$. Under what condition on $T$ will all frequencies of $x(t)$ be retained in $x(nT)$?~~

~~6) Are the following systems with or without memory, causal or non-causal?~~
~~a. $y(t) = 5u(t)$~~
~~b. $y(t) = \sin u(t) + \sin u(t+1)$~~
~~c. $y(t) = (\sin t)u(t-1)$~~

d. $\frac{d^2 y(t)}{dt^2} = u(t)$

e. $y(t) = \int_0^t 3t^2 u(\tau)\, d\tau$