University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 3

Issued: September 10, 2007 Due: September 17, 2008 (In class)

Reading Assignments:

Read Chapter 2 (except 2.9) of Chen

Paper and Pencil Assignments:

1) Problem 1.31: Consider $\cos\omega_k n$ with T=1 and

$$\omega_k = \frac{2\pi k}{N}$$

where N is a given positive integer and k is an integer ranging from -∞ to ∞. How many different $\cos\omega_k n$ are there (in terms of N)? What are their frequencies (in terms of N)?

2) Problem 1.35: What are the sample sequences of $x(t) = \cos 50t + 2\sin 70t$ for $T = \pi/45$, $\pi/30$ and $\pi/180$. Under what condition on T will all frequencies of $x(t)$ be retained in $x(nT)$?

3) Problem 2.3: Consider a CT system whose input and output are related by

$$y(t) = \begin{cases} u^2(t)/u(t-1) & \text{if } u(t-1) \neq 0 \\ 0 & \text{if } u(t-1) = 0 \end{cases}$$

Show that the system satisfies the homogeneity property but not the additivity property. Thus the homogeneity property does not imply the additivity property.

4) Problem 2.4: Show that if $\{u_1 + u_2\} \rightarrow \{y_1 + y_2\}$, then

$$\left\{\frac{a}{b}u_1\right\} \rightarrow \left\{\frac{a}{b}y_1\right\}$$

for any integers *a* and *b*. Thus the additivity property implies $\{\alpha u_1\} \rightarrow \{\alpha y_1\}$ for any rational number α. In other words, the additivity property *almost* implies the homogeneity property.

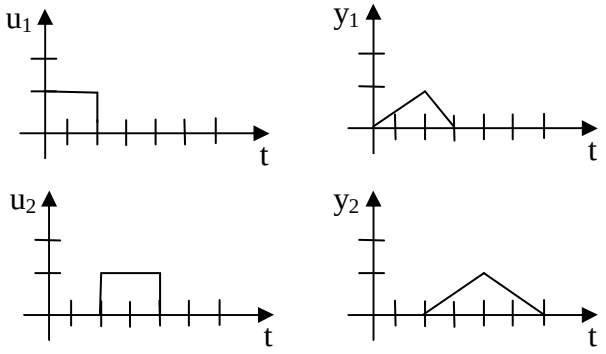
5) Problem 2.5:Discuss whether or not each of the following equations is memoryless, linear, time-invariant, and casual:
   a. $y(t) = -2 + 3u(t)$
   b. $y(t) = \sqrt{u(t)}$
   c. $y(t) = u(t)u(t-1)$

d. $y(t) = tu(t)$

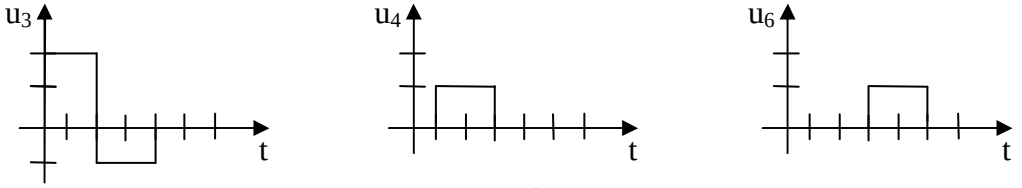
e. $y(t) = \int_{t_0}^{t} u(\tau)\, d\tau + y(t_0)$

f. $y(t) = \int_{t_0}^{t} \pi u(\tau)\, d\tau + y(t_0)$

6) Problem 2.7: Consider a CT linear system. Its zero-state responses excited by u_1 and u_2 are shown in the following figure (a). Is the system time-varying or time-invariant? Can you find the zero-state responses excited by the inputs u_3, u_4, and u_5 shown in figure (b)?

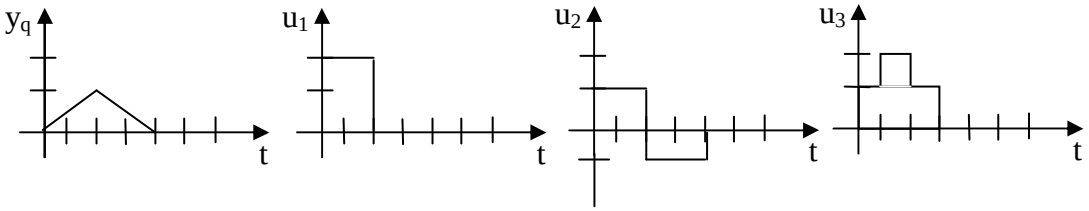


(a)



(b)

7) Problem 2.9: Consider a CT LTI system. Suppose its step response (the zero-state response excited by a step input or $u(t) = q(t)$ is as shown in Figure (a). Find the outputs excited by the inputs shown in Figure (b) through (d).



(a) (b) (c) (d)