University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 4

Issued: September 17, 2007 Due: September 24, 2008 (In class)

Reading Assignments:

Read Chapter 3 of Chen.

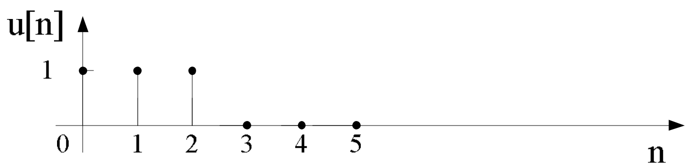
Computer Assignments:

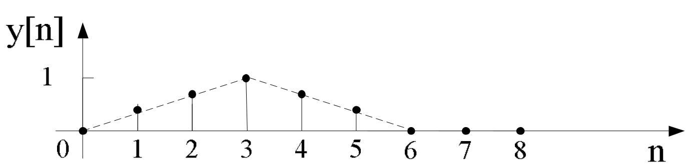
1) Explore the "*Joy of Convolution*" and "*Joy of Convolution (Discrete Time)*" sections at http://www.jhu.edu/~signals.

If the Java applets in the above webpages are not running properly, install the latest Java Runtime Environment (JRE) from http://java.sun.com/javase/downloads/index.jsp

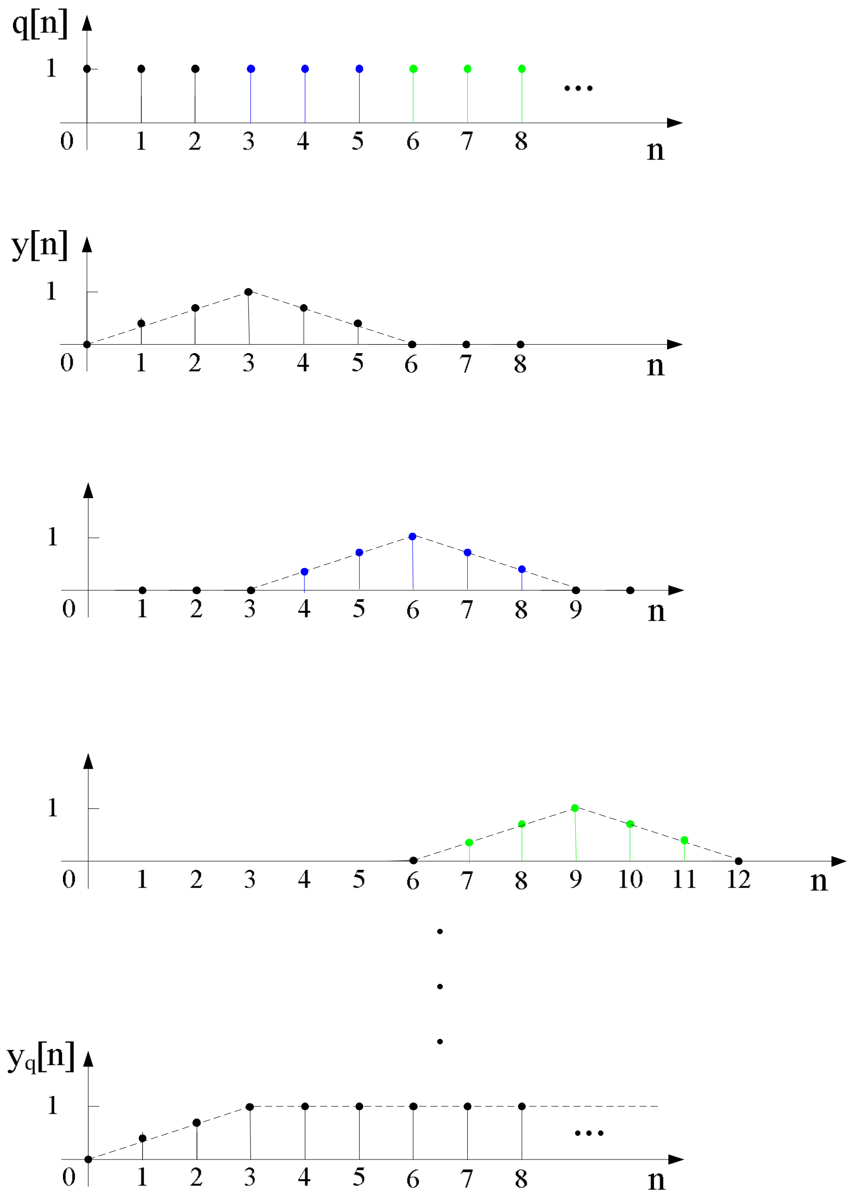
Paper and Pencil Assignments:

1) Problem 2.16: Consider a DT LTI system with the input-output pair shown below. What is its step response (the output excited by the step sequence q[n])? What is its impulse response (the output excited by the impulse sequence δ[n])?

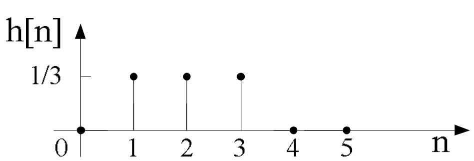




Solution:



Because $\delta[n] = q[n] - q[n-1]$, the impulse response $h[n] = y_q[n] - y_q[n-1]$



2) Problem 3.2: Consider the sequence h=[1 2 3 -2] which is located at n=0:3 and u[1 0 -2 4 4 5] which is located at n=0:5. Compute their convolution for n=-2:12.

Solution:
The result is [0 0 $\underline{1}$ 2 1 -2 6 29 14 7 -10 0 0], where the underlined number indicates n=0.

3) Problem 3.3: Consider the polynomials

$$h(s) = s^3 + 2s^2 + 3s - 2$$

And

$$u(s) = s^5 - 2s^3 + 4s^2 + 4s + 5$$

Compute h(s)u(s). Verify that its coefficients are the same as those in Problem 2. Thus the multiplication of two polynomials can be computed as the convolution of their coefficients.

Solution:

$$h(s)u(s) = s^8 + 2s^7 + s^6 - 2s^5 + 6s^4 + 29s^3 + 14s^2 + 7s - 10$$

Its coefficients are the same as the convolution in Prob. 3.2 for n=0:8, thus the product of two polynomials can be computed using a convolution.

In MATLAB, typing

```
h=[1 2 3 -2];
u=[1 0 -2 4 4 5];
conv(h,u)
```

yields

```
1     2     1    -2     6    29    14     7   -10
```

4) Problem 3.6: Consider a DT LTI system with impulse response h[0]=0 and h[n]=1 for all n≥1. Find a difference equation to describe the system.

Solution:

$$y[n] = \sum_{k=0}^{n} h[n-k]u[k] = \sum_{k=0}^{n-1} u[k]$$

This is a general equation that holds for all $n \geq 0$. Recall that the equation describes only the zero-state response or, equivalently, $y[n] = u[n] = 0$, for $n < 0$.

Because

$$y[n+1] = \sum_{k=0}^{n} u[k] = \sum_{k=0}^{n-1} u[k] + u[n]$$
$$= y[n] + u[n]$$

we have

$$y[n+1] - y[n] = u[n]$$

5) Problem 3.9: Consider the difference equation

$$y[n] + 2y[n-1] = u[n-1] + 3u[n-2] + 2u[n-3]$$

Does it describe a causal system? What is its order?

Solution:
Because the output $y[n]$ depends on past input $(u[n-1], u[n-2], u[n-3])$ and past output $y[n-1]$, it is causal. Its order is 3.

6) Problem 3.10: Compute the impulse response of the DT system in Problem 5. Is it FIR or IIR? If it is FIR, what is its length? Can you find a non-recursive difference equation to describe the system?

Solution:
If $u[n]=\delta[n]$, then $h[n]=y[n]$. We write
$h[n]=-2h[n-1]+\delta[n-1]+3\delta[n-2]+2\delta[n-3]$
with $h[n]=0$ for $n<0$.
n=0: $h[0]=-2h[-1]+\delta[-1]+3\delta[-2]+2\delta[-3]=0$
n=1: $h[1]=-2h[0]+\delta[0]+3\delta[-1]+2\delta[-2]=1$
n=2: $h[2]=-2h[1]+\delta[1]+3\delta[0]+2\delta[-1]=-2+3=1$
n=3: $h[3]=-2h[2]+\delta[2]+3\delta[1]+2\delta[0]=-2\times1+2=0$
n=4: $h[4]=-2h[3]+\delta[3]+3\delta[2]+2\delta[1]=0$
$h[n]=0$ for $n\geq 3$.
It is FIR with length 3. Because $h[1]=h[2]=1$ and the rest zero, the convolution
$y[n]=\sum_{k=0}^{n}h[k]u[n-k]$ reduces to
$$y[n]=h[0]u[n]+h[1]u[n-1]+h[2]u[n-2]+h[3]u[n-3]+\cdots$$
$$=u[n-1]+u[n-2]$$
This is a delayed-form difference equation of order 2. It is a nonrecursive difference equation.

$$y_2(t)=\begin{cases} 0.5t^2, \ 0\leq t\leq 2 \\ -(t-3)^2+3, 2\leq t\leq 4 \\ 0.5(t-6)^2, 4\leq t\leq 6 \\ 0, \ otherwise \end{cases}$$