

University of Kentucky  
Department of Electrical and Computer Engineering

**EE421G: Signals and Systems I – Fall 2007**

Problem Set 6

Issued: October 4, 2007

Due: October 10, 2007 (In class)

**Reading Assignments:**

Read Chapter 3.7, 3.8, 4.1, 4.2, 4.3 of Chen

**Computer Assignments:**

- 1) Simulating a discrete-time LTI system in Matlab is easy. During lecture, you have learned that a causal difference equation can be written as follows:

$$a_1 y[n] + a_2 y[n-1] + \dots + a_{M+1} y[n-M] = b_1 u[n] + b_2 u[n-1] + \dots + b_{N+1} u[n-N]$$

Given a finite duration signal  $u[n]$  (specified as a vector in Matlab), you can compute the zero-state output of the system using the command `filter`:

```
>> y = filter(b, a, u)
```

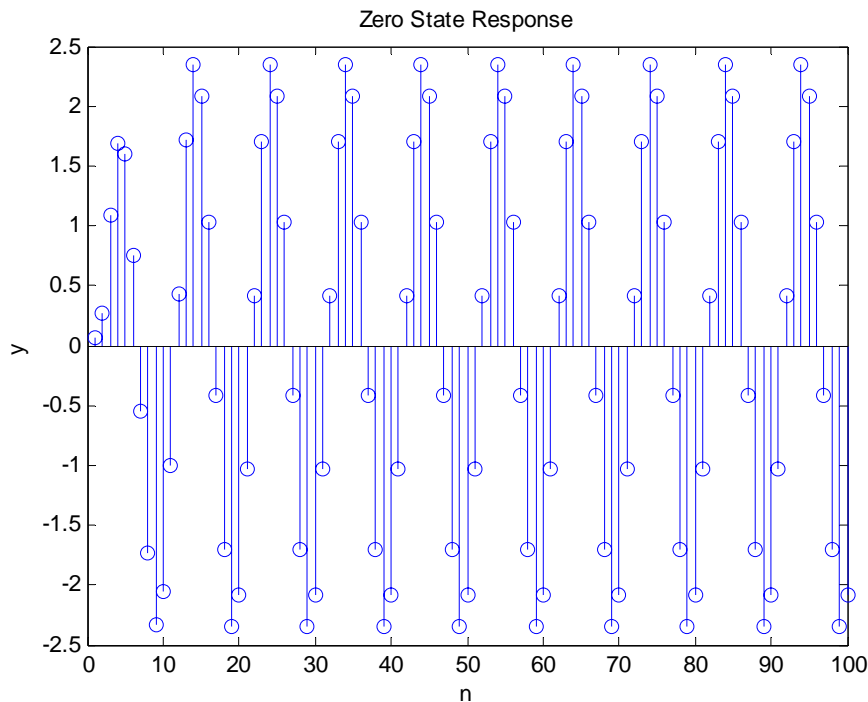
where  $a$  is the vector  $[a_1, a_2, \dots, a_{M+1}]$  and  $b$  is the vector  $[b_1, b_2, \dots, b_{N+1}]$ .

- a) Assuming zero initial conditions and input  $u[n] = \cos(\frac{\pi}{5}n)q[n]$ , compute the first 100 values of the output of the following differential equation:

$$y[n] - 1.143y[n-1] + 0.4128y[n-2] = 0.0675u[n] + 0.1349u[n-1] + 0.675u[n-2]$$

MATLAB code:

```
b = [0.0675, 0.1349, 0.675];  
a = [1, -1.143, 0.4128];  
n = 0:100;  
u = cos(pi/5*n);  
ya = filter(b, a, u);  
figure(1);  
stem(ya);  
xlabel('n');  
ylabel('y');  
title('Zero State Response');  
set(gca, 'Xlim', [0, 100]);
```



- b) Alternatively, you can first compute the impulse response of the LTI system using the following command:

```
>> [h, t] = impz(b, a, n)
```

where  $n$  is the number of impulse response values to be computed. The vector  $h$  contains the values of the impulse response, and  $t$  contains the corresponding time indices. After computing  $h[n]$ , you can then use the `conv` command between  $h[n]$  and the input  $u[n]$  to compute the output. Repeat part a) using this method and plot the outputs for both part a) and b) versus time using the command `stem`. Explain any discrepancies, if any. Remember, you can always use

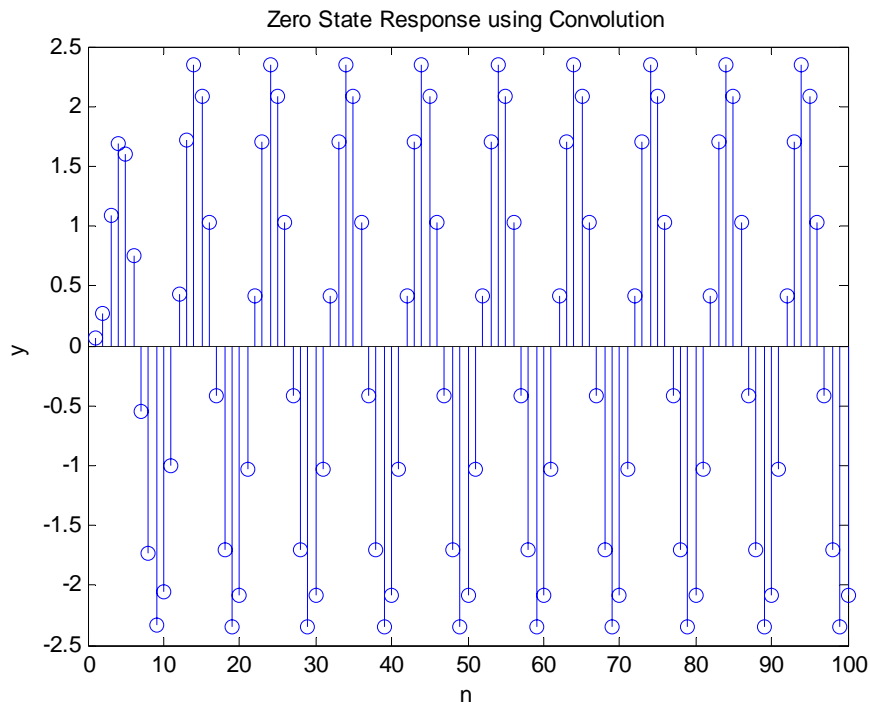
```
>> help <command name>
```

to find out the proper syntax and usage of any command

MATLAB code:

```
[h, t] = impz(b, a, 300);
yb = conv(h, u);
figure(2);
stem(yb);
xlabel('n');
ylabel('y');
```

```
title('Zero State Response using Convolution');
set(gca, 'Xlim', [0, 100]);
```



- 2) Continuous-time LTI system is handled a little differently in Matlab. Given the differential equation

$$a_1 \frac{d^M}{dt^M} y(t) + \dots + a_M \frac{d}{dt} y(t) + a_{M+1} y(t) = b_1 \frac{d^N}{dt^N} u(t) + \dots + b_N \frac{d}{dt} u(t) + b_{N+1} u(t)$$

you can specify the CT LTI system as a transfer function object as follows:

```
>> sys = tf(b, a)
```

where  $a$  is the vector  $[a_1, a_2, \dots, a_{M+1}]$  and  $b$  is the vector  $[b_1, b_2, \dots, b_{N+1}]$ . Matlab cannot directly compute the output of a CT system because one cannot specify a continuous-time signal in a digital computer. All Matlab can do is to compute the output at specified time instances using the command `lsim`:

```
>> y = lsim(sys,u,t)
```

where  $sys$  is the transfer object function,  $t$  is a vector that contains the discrete time instances,  $u$  is the vector of the input signal sampled at those time instances and  $y$  is the output sampled at the same time instances.

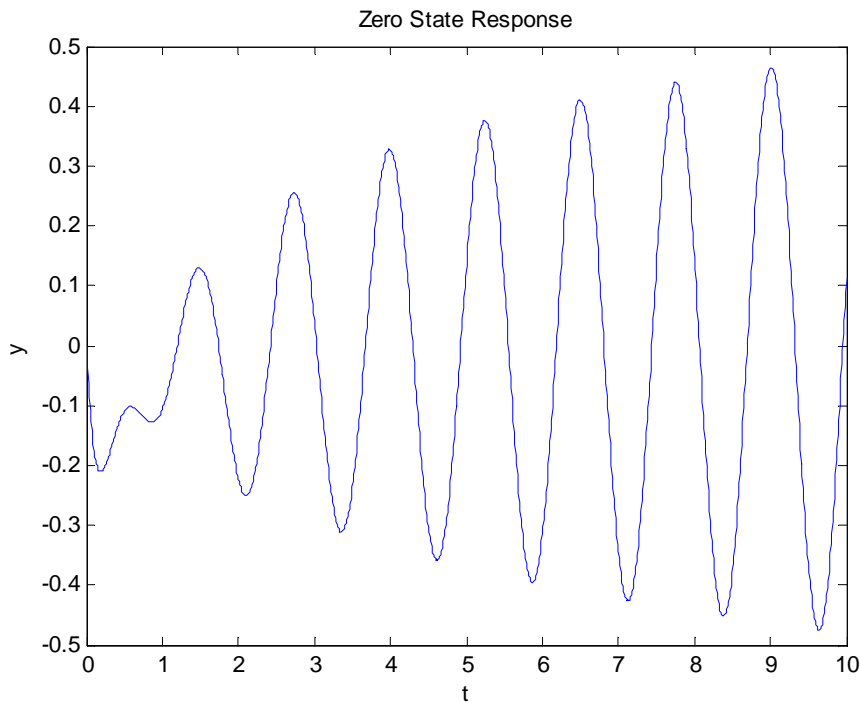
Using these command, compute the zero-state output response of the following differential equation

$$\frac{d^3}{dt^3} y(t) + 6 \frac{d^2}{dt^2} y(t) + 11 \frac{d}{dt} y(t) + 6y(t) = \frac{d^2}{dt^2} u(t) + 3 \frac{d}{dt} u(t) + 4u(t)$$

for time instances starting from 0.01 sec to 10 sec at an internal of 0.01 sec. The input is  $u(t) = \log(2t) \cos(5t)$ .

MATLAB code:

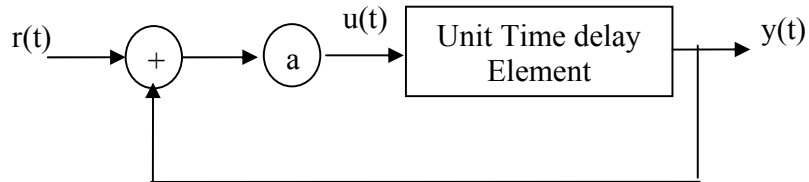
```
t = 0.01:0.01:10;
u = log(2*t).*cos(5*t);
b = [1, 3, 4];
a = [1, 6, 11, 6];
sys = tf(b, a);
y = lsim(sys, u, t);
plot(t, y);
xlabel('t');
ylabel('y');
title('Zero State Response');
```



**Paper and Pencil Assignments:**

**The first three are the same as last homework. You do not need to turn them in if you have done them previously. BUT DO NOT FORGET THE REMAINING THREE PROBLEMS!**

- 1) Problem 3.14: Consider the positive feedback system show below. Find its impulse response.

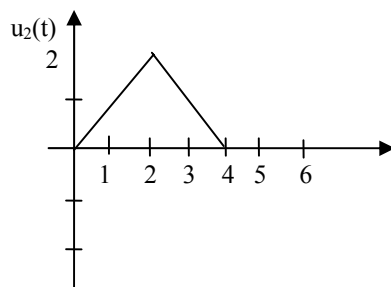
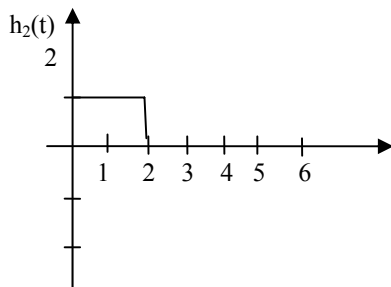
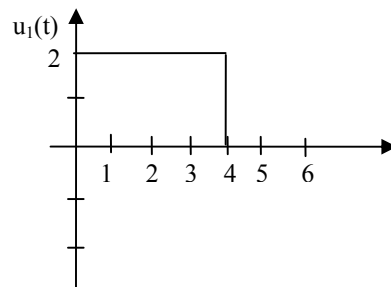
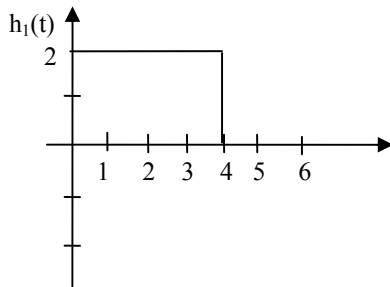


Solution:

$$h(t) = a\delta(t-1) - a^2\delta(t-2) + a^3\delta(t-3) - \dots$$

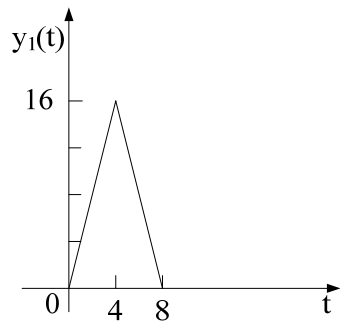
$$= \sum_{k=1}^{\infty} (-1)^{k-1} a^k \delta(t-k)$$

- 2) Problem 3.15: Compute the integral convolution of  $h_1(t)$  and  $u_1(t)$ , shown below.



Solution:

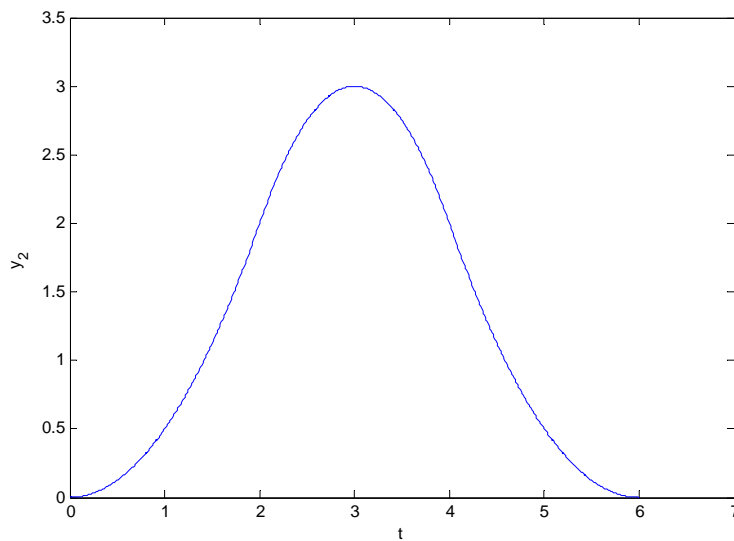
Part 1:



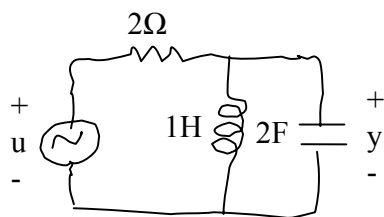
Part 2:

$$y_2(t) = \begin{cases} 0.5t^2, & 0 \leq t \leq 2 \\ -(t-3)^2 + 3, & 2 \leq t \leq 4 \\ 0.5(t-6)^2, & 4 \leq t \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

$y_2(1) = 0.5, y_2(2) = 2, y_2(3) = 3$



3) Problem 3.18: Find a differential equation to describe the following network.



Solution:

$$2\dot{y} + \int_0^t y(\tau)d\tau = \frac{u(t) - y(t)}{2}$$

$$\Rightarrow 4\ddot{y}(t) + 2y(t) = \dot{u}(t) - \dot{y}(t)$$

$$\text{differential equation: } 4\ddot{y}(t) + \dot{y}(t) + 2y(t) = \dot{u}(t)$$

- 4) Problem 4.1: What is the fundamental frequency and fundamental period of the signal  
Express it in complex Fourier series and plot the magnitudes and phases of its frequency components.

$$x(t) = 3 + \sin 6t - 2 \cos 6t + \pi \sin 9t - \cos 12t$$

Solution:

The gcd of {6, 9, 12} is 3. Thus the fundamental frequency is  $\omega_0 = 3 \text{ rad} / \text{s}$  and the

fundamental period is  $P = \frac{2\pi}{3}$ .

$$x(t) = 3 + \frac{e^{j6t} - e^{-j6t}}{2j} - 2 \cdot \frac{e^{j6t} + e^{-j6t}}{2} + \pi \cdot \frac{e^{j9t} - e^{-j9t}}{2j} - \frac{e^{j12t} + e^{-j12t}}{2}$$

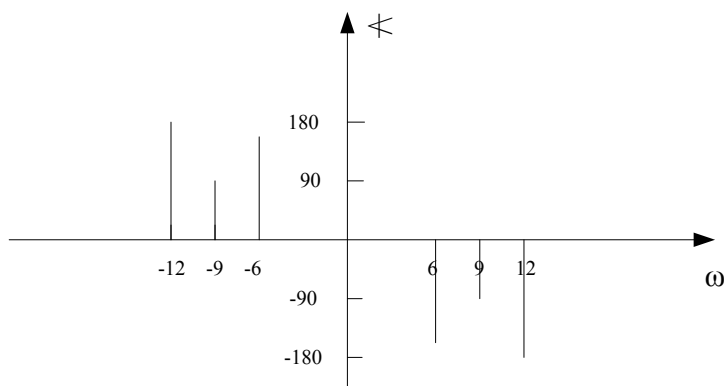
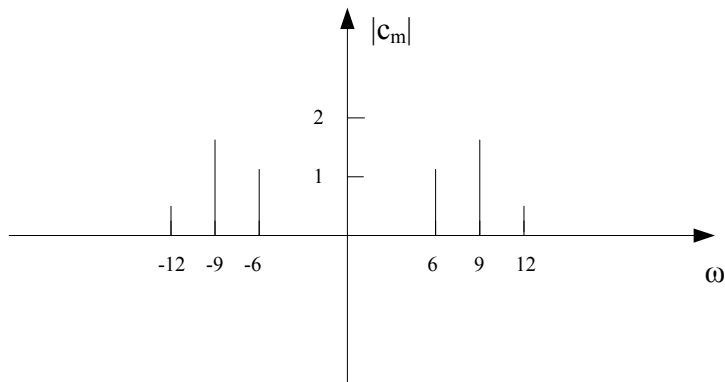
$$= 3 + (-1 - j0.5)e^{j6t} + (-1 + j0.5)e^{-j6t} + (-0.5\pi j)e^{j9t} + 0.5\pi j e^{-j9t} - 0.5e^{j12t} - 0.5e^{-j12t}$$

Because  $-1 + j0.5 = 1.12e^{j2.68} = 1.12e^{j153.4^\circ}$  and  $-1 - j0.5 = 1.12e^{-j153.4^\circ}$

We have

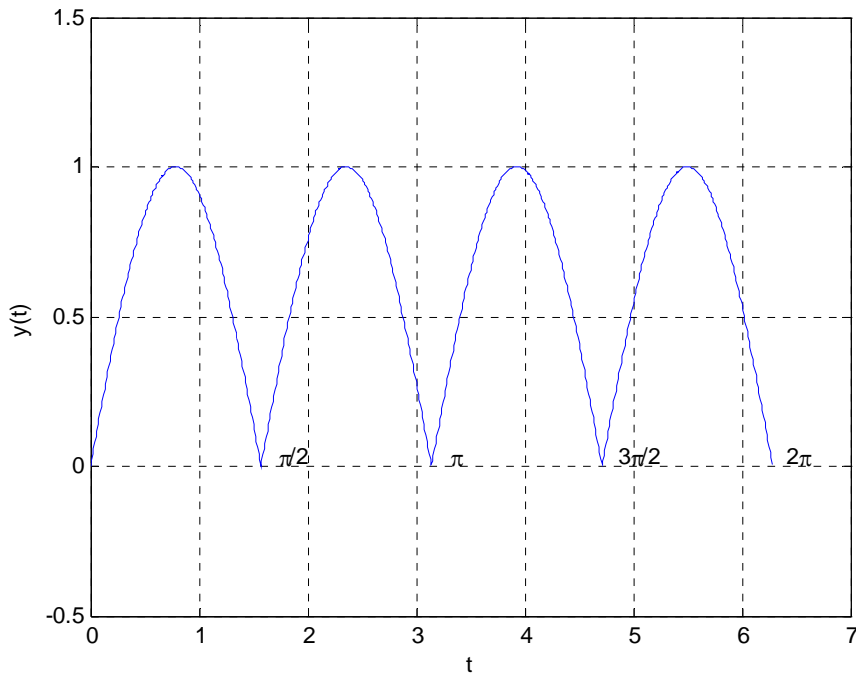
$$x(t) = 3e^{j0} + 1.12e^{-j153.4^\circ} e^{j6t} + 1.12e^{j153.4^\circ} e^{-j6t} + 1.57e^{-j90^\circ} e^{j9t} + 1.57e^{j90^\circ} e^{-j9t}$$

$$+ 0.5e^{-j180^\circ} e^{j12t} + 0.5e^{j180^\circ} e^{-j12t}$$



- 5) Problem 4.2: Consider the full-wave rectifier in Figure 2.21(a). What is its output  $y(t)$  if the input is  $u(t)=\sin 2t$ ? What are the fundamental period and fundamental frequency of  $y(t)$ ? Express the output in Fourier series. Does the output contain frequency other than 2 rad/s?

Solution:



$$\text{Fundamental period} = \frac{\pi}{2}$$

$$\text{Fundamental frequency} = \frac{2\pi}{\pi/2} = 4 \text{ rad/s} = \omega_0$$

$$\begin{aligned} c_m &= \frac{2}{\pi} \int_0^{\pi/2} \sin 2t e^{-j4mt} dt \\ &= \frac{2}{\pi} \left[ \frac{e^{-j4mt} (-j4m \sin 2t - 2 \cos 2t)}{4 + (-j4m)^2} \right] \Bigg|_{t=0}^{\pi/2} \\ &= \frac{2}{\pi} \frac{2e^{-j2\pi m} + 2}{4 - 16m^2} = \frac{2 \times 4}{4\pi(1 - 4m^2)} \\ &= \frac{2}{\pi(1 - 4m^2)} \end{aligned}$$

$$\therefore y(t) = \sum_{m=-\infty}^{\infty} \frac{2}{\pi(1 - 4m^2)} e^{j4mt}$$

It contains frequency  $4m$ , for all integer  $m$ .

- 6) Problem 4.5: What is the total energy of the signal in Problem 4? What is its average power in one period? How many percentage of the average power lies inside the frequency range  $[-7, 7]$ ?

Solution:

Its total energy is infinite.

Its total average power is

$$\begin{aligned}\sum |c_m|^2 &= 3^2 + 2 \times (1.12)^2 + 2 \times (1.57)^2 + 2 \times (0.5)^2 \\ &= 9 + 2(1.25) + 2(2.46) + 0.5 = 16.92\end{aligned}$$

The average power inside  $[-7, 7]$  is  $9 + 2(1.25) = 11.5$

It is  $\frac{11.5}{16.92} = 0.68 = 68\%$  of the total average power.