University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 7

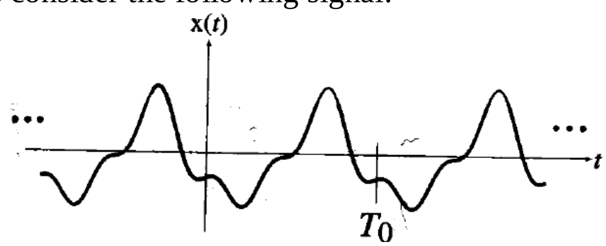
Issued: October 13, 2007 Due: October 19, 2007 (In class)

Reading Assignments:

Read Chapter 4 of Chen

Computer Assignment:

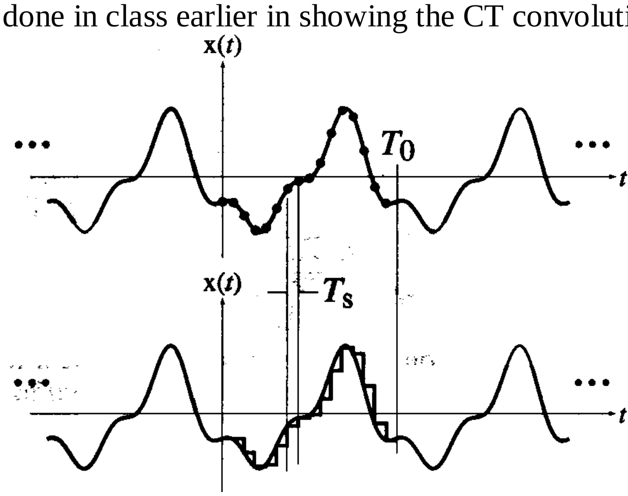
In this problem, we learn how to numerically compute the Fourier series of an arbitrary periodic signal. Let's consider the following signal:



This signal presents some problems. It is not at all obvious how to describe it. Up to this time in our study, we needed a mathematical description of a signal so that we can symbolically compute its Fourier series coefficient:

$$c_k = \frac{1}{T_0}\int_0^{T_0} x(t)\exp(-jk\omega_0 t)\,dt$$

There is a better way – if we have a set of N samples of the signal taken from one period, we can estimate c_k numerically. The larger the N, the better the approximation becomes. Let's say, the sampling interval is T_S. We can approximate the signal as a piece-wise flat function as we have done in class earlier in showing the CT convolution formula.



The Fourier series coefficient can then be approximated as a sum of the area of rectangles:

$$\begin{aligned} c_k &\approx \frac{1}{T_0}\sum_{n=0}^{N-1} x(nT_S)\exp(-jk\omega_0 nT_S)T_S \\ &= \frac{1}{N}\sum_{n=0}^{N-1} x(nT_S)\exp\left(-j\frac{k2\pi}{N}\right) \end{aligned}$$

The last line uses the simple relationship $T_0=NT_S$ (N samples in one period) and $\omega_0 = 2\pi/T_0 = 2\pi/(NT_S)$. The summation $\sum_{n=0}^{N-1} x(nT_S)\exp\left(-j\frac{k2\pi}{N}\right)$ turns out to be a very important Fourier representation called Discrete Fourier Transform (DFT), and it can be efficiently computed using Fast Fourier Transform (FFT). Thus, if `x` is a N-sample version of x(t) in MATLAB, we can use the following to compute the Fourier series coefficient:

```
>> c = fft(x)/length(N)
```

As it turns out, only the first half of the vector c represents the first N/2 Fourier series coefficients of x(t). The remaining half represents a reflected version of the conjugate of the same set of coefficients which we can ignore.

Okay, it's your turn to do something. Use the above method to find the magnitude and phase spectrums of the first 64 Fourier series coefficients of the periodic signal x(t), one period of which is described by

$$x(t) = \sqrt{1-t^2} \quad -1 < t < 1$$

Here are some hints to get you started. The fundamental period of the signal is 2. Use N=128 to sample the signal between 0 and 2. To do that, we first define the time instances to do the sampling:

```
>> n = 0:127
>> Ts = 2/128;
>> t = Ts*n;
```

Then, we can do the sampling.

```
>> x = [sqrt(1-t(1:64).^2) sqrt(1-(t(65:128)-2).^2)];  % Can
you explain?
```

Finally, apply the FFT and use the stem plots to show the magnitude (with command `abs`) and phase (with command `angle`) of the harmonics.

Paper and Pencil Assignments:

1) Problem 4.9
2) Problem 4.10
3) Problem 4.11
4) Problem 4.12
5) Problem 4.15
6) Problem 4.16
7) Problem 4.19