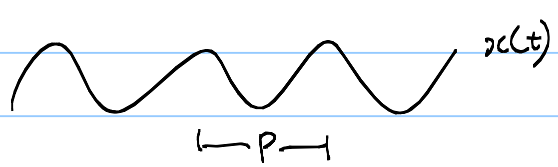
Note Title 10/19/2007

4.9



$$\omega_0 = \frac{2\pi}{P} \qquad x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-jk\omega_0 t}$$

signal is also period with "period" 2P

$$\omega_1 = \frac{2\pi}{2P} \qquad x(t) \overset{?}{=} \sum_{k=-\infty}^{\infty} d_k e^{-jk\omega_1 t}$$

Harmonics $0, \pm\omega_1, \pm 2\omega_1, \pm 3\omega_1, \pm 4\omega_1, \ldots$

Since $\omega_0 = 2\omega_1$ $0, \quad , \pm\omega_0, \quad , \pm 2\omega_0, \ldots$

One possible d_k's will be

$$d_k = C_{k/2} \quad \text{for even } k$$
$$d_k = 0 \quad \text{for odd } k$$

ONLY REP.

Is this the only possible d_k's ?

$\{\vec{e}_i, \vec{e}_j, \vec{e}_k\}$ $\quad \vec{u} = 3\vec{e}_i + 0\,\vec{e}_j + 4\vec{e}_k$

$$= a\cdot\vec{e}_i + b\cdot\vec{e}_j + c\,\vec{e}_k$$

where $a \neq 3$, $b \neq 0$, $c \neq 4$

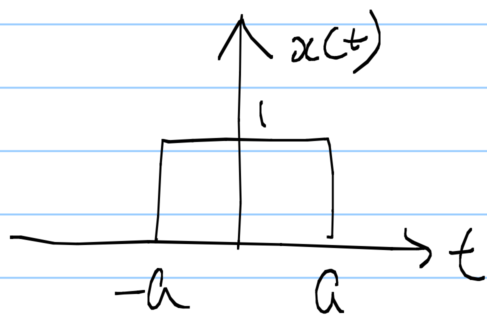
Answer. No $\because$ $\langle \vec{u}, \vec{e}_i \rangle = \langle a\vec{e}_i + b\vec{e}_j + c\vec{e}_k, \vec{e}_i \rangle$

$$= a\langle \vec{e}_i, \vec{e}_i \rangle + b\cancel{\langle \vec{e}_j, \vec{e}_i \rangle}^{0} + c\cancel{\langle \vec{e}_k, \vec{e}_i \rangle}^{0}$$

$$= a = 3$$

4.10

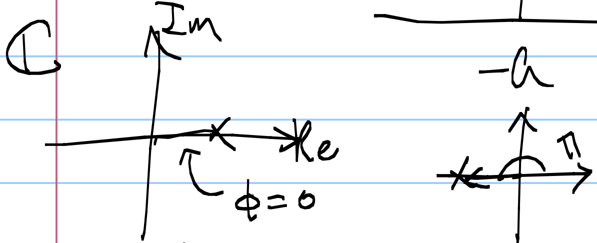
4.33.



$\rightarrow X(\omega) = \dfrac{2\sin(a\omega)}{\omega}$

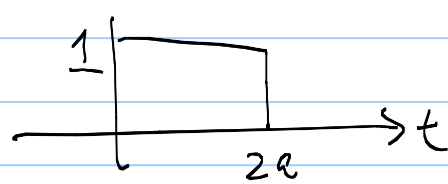
$\downarrow$

Magnitude, Phase



The magnitude & phase plots in Ex 4.3.3

one for



4.11 frequency spectrum = Fourier Transform $X(\omega)$

Do the integration

$$\boxed{\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \\ x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \end{aligned}}$$ ✓

$x(t) = e^{-2t} \quad t = (-\infty, \infty)$

$$X(\omega) = \int_{-\infty}^{+\infty} e^{-2t}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(-2-j\omega)t}\, dt$$

$$= \frac{1}{-2-j\omega}\left[e^{(-2-j\omega)t} \right]_{-\infty}^{\infty}$$

upper limit $= \lim_{t\to\infty} e^{(-2-j\omega)t} = \lim_{t\to\infty} e^{-2t} \cdot e^{-j\omega t}$

$$= \lim_{t\to\infty} \underbrace{e^{-2t}}_{\to 0} \overbrace{\left[\cos(\omega t) - j\sin(\omega t)\right]}^{\text{oscillating}}$$

$|\cdot| = 1$

$= 0$

$1 \cdot 1 = 1$

lower limit $= \lim_{t \to -\infty} e^{-2t} [\cos(\omega t) - j\sin(\omega t)]$

∞

$= \infty$ ← Does not exist

$X(\omega)$ does not exist !

~~4.12~~ Magnitud, phase spectrum (MATLAB)
abs( ) angle( )

$\boxed{4.12}$ Write them out and stare at them

$\boxed{4.15}$ $x(t) = e^{-0.1t} \sin(10t + \frac{\pi}{4})\, q(t)$

$$\downarrow$$

$$\frac{1}{2j}\Big[\underbrace{e^{j(10t+\frac{\pi}{4})}} - e^{-j(10t+\frac{\pi}{4})}\Big]$$

$$\downarrow$$

$$e^{j10t} \cdot e^{j\frac{\pi}{4}}$$

Repeat just like 4.11

$$= 2\pi \cdot \text{Energy}(x(t)) \,?$$

4.16 $\left\{ \; L \cdot B \geq \pi \right.$

$$L = \frac{\left[\int_{-\infty}^{\infty} |x(t)|\, dt\right]^2}{\left[\int_{-\infty}^{\infty} |x(t)|^2 dt\right]} \qquad B = \frac{\left[\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega\right]}{2|X(0)|^2}$$

$$\int_{-\infty}^{\infty} x(t)x^*(t)\, dt$$

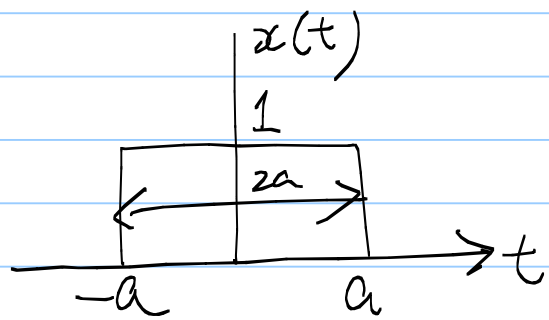
$$\shortparallel$$

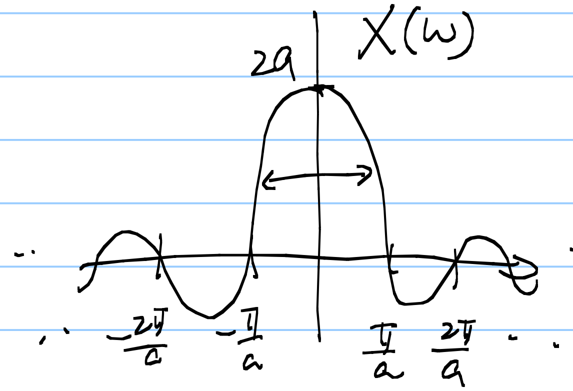
$$\text{Energy}(x(t))$$

Parseval Relation

$$\text{Energy}(x(t)) = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \cdot d\omega$$

Ex





$$L = \frac{4a^2}{2a} \qquad B = \frac{\cancel{2\pi} \cdot \cancel{2a}}{2\,4a^2}$$

$$= 2a \qquad\qquad = \frac{\pi}{2a}$$

$$L \cdot B = \pi$$

How to prove $L \cdot B \geq \pi$ for general $x(t)$, $X(\omega)$?

$$L \cdot B = \frac{\left[\int |x(t)|\, dt\right]^2}{\cancel{\text{Energy}}} \cdot \frac{\cancel{2}\pi \cdot \cancel{\text{Energy}}}{\cancel{2}|X(0)|^2}$$

$$= \pi \underbrace{\frac{\left[\int |x(t)|\, dt\right]^2}{|X(0)|^2}}_{\Delta}$$

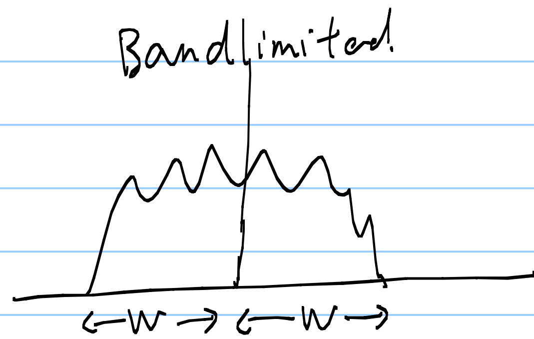
if $L \cdot B \geq \pi$, $\Delta \geq 1$ What is $X(0)$?

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\omega = 0 \Rightarrow X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{Area under } x(t)$$

$\boxed{4.19}$

$$x(t) \longleftrightarrow X(\omega)$$



$$\downarrow$$

$$x_m(t) = x(t)\cos(\omega_c t) \longrightarrow X_m(\omega) = \int_{-\infty}^{\infty} x(t)\left[\frac{1}{2}e^{j\omega_c t} + \frac{1}{2}e^{-j\omega_c t}\right] e^{-j\omega t}\, dt$$

$$X_m(\omega) = \int_{-\infty}^{\infty} \frac{1}{2} x(t)\, e^{-j(\omega-\omega_c)t}\, dt + \int_{\infty}^{\infty} \frac{1}{2} x(t)\, e^{-j(\omega+\omega_c)t}\, dt$$

$$= \frac{1}{2} X(\omega-\omega_c) + \frac{1}{2} X(\omega+\omega_c)$$

Compare w/ $X(\omega)$