University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 8

Issued: October 23, 2007 Due: October 26, 2007 (In class)

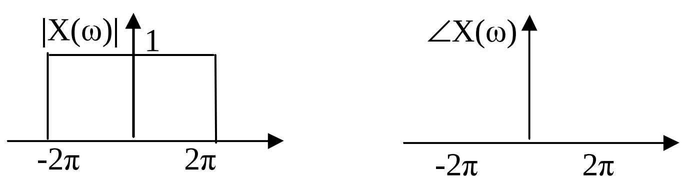
There are five pencil and paper problems in this assignment. They are based on various properties of Fourier Transform and commonly-used transform pairs, which are listed at the end of this assignment for your reference. Please read them over before starting the assignment.

1) (1 point) Given a LTI system L[•], a signal v(t) is the *eigen-signal* of the system if, using v(t) as input, the zero-state output is a scaled version of the input, i.e.

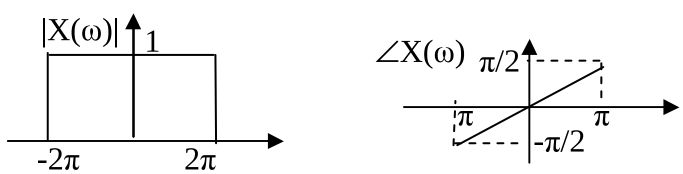
$$L[v(t)] = \lambda \cdot v(t)$$

where λ, the eigen-value of v(t), is a complex constant that depends on v(t). Show that if h(t) is the impulse response of L[•], then for any complex value ω, $v(t) = e^{j\omega t}$ is an eigen-signal of L[•] and the corresponding eigen-value is H(ω), the Fourier Transform of h(t) evaluated at ω. Hint: Start with the convolution formula: $y(t) = \int_{-\infty}^{\infty} h(\tau) v(t-\tau) d\tau$.

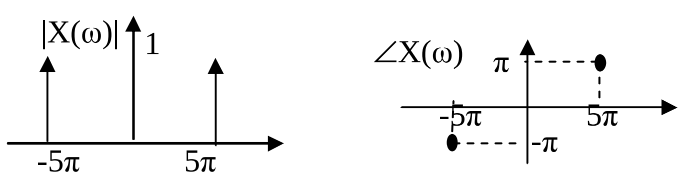
2) (3 points) Graph the inverse CTSTs of the following four pairs of magnitude and phase responses:



a.



b.



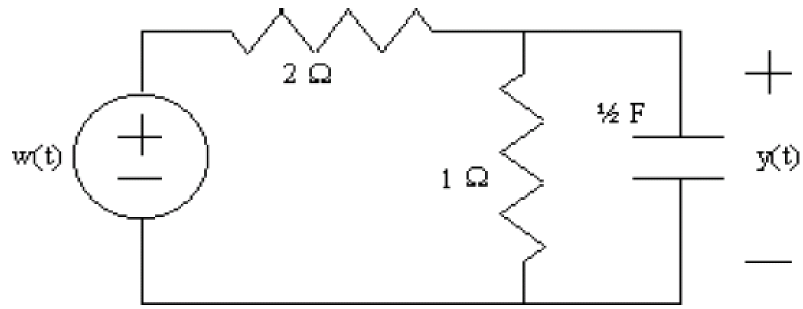
c.

3) (2 points) Using the multiplication-convolution duality of the Fourier Transform, find an expression for y(t) that does not use the convolution operator *:

a. y(t) = rect(t) * cos(πt)

b. $\mathcal{F}$[y(t)] = $\text{sinc}^2(\omega/2)$

4) (3 point) Given the following circuit



a. Write down the differential equation that governs the input w(t) and y(t).
b. By applying Fourier Transform directly onto the differential equation, compute the Fourier transform of the impulse response of the circuit $H(\omega) = Y(\omega)/W(\omega)$.
c. Using part b), compute the zero-state output y(t) if $w(t) = e^{-3t}q(t)$.

5) (1 point) One major problem in real instrumentation systems is electromagnetic interference caused by the 60Hz power lines. A system with an impulse response of the form $h(t) = A[q(t) - q(t - t_0)]$ can reject 60 Hz and all its harmonics. Find the numerical value of t_0 that makes this happen.

TABLE 5.1 Fourier Transform Properties

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Convolution	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
Differentiation	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$

TABLE 5.2 Fourier Transform Pairs

Time Domain Signal	Fourier Transform
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$
$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$	$F(\omega)$
$\delta(t)$	1
$A\delta(t - t_0)$	$Ae^{-j\omega t_0}$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
1	$2\pi\delta(\omega)$
K	$2\pi K\delta(\omega)$
$\text{sgn}(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$\text{rect}(t/T)$	$T\text{sinc}(\omega T/2)$
$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
$\text{rect}(t/T)\cos(\omega_0 t)$	$\frac{T}{2}\left[\text{sinc}\left(\frac{(\omega - \omega_0)T}{2}\right) + \text{sinc}\left(\frac{(\omega + \omega_0)T}{2}\right)\right]$
$\frac{\beta}{\pi}\text{sinc}(\beta t)$	$\text{rect}(\omega/2\beta)$
$\text{tri}(t/T)$	$T\,\text{sinc}^2(T\omega/2)$
$\text{sinc}^2(Tt/2)$	$\frac{2\pi}{T}\text{tri}(\omega/T)$
$e^{-at}u(t)$, $\text{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
$te^{-at}u(t)$, $\text{Re}\{a\} > 0$	$\left(\frac{1}{a + j\omega}\right)^2$
$t^{n-1}e^{-at}u(t)$, $\text{Re}\{a\} > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$
$e^{-a\|t\|}$, $\text{Re}\{a\} > 0$	$\frac{2a}{a^2 + \omega^2}$
$\sum_{n=-\infty}^{\infty} g(t - nT_0)$	$\sum_{n=-\infty}^{\infty} \omega_0 G(n\omega_0)\delta(\omega - n\omega_0)$, $\omega_0 = \frac{2\pi}{T_0}$
$\sum_{n=-\infty}^{\infty} g(t - nT_0) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$	$H(\omega) = \|H(\omega)\|\angle\phi(\omega) = \frac{\|V_2(\omega)\|\angle V_2}{\|V_1(\omega)\|\angle V_1} = \frac{\|V_2(\omega)\|}{\|V_1(\omega)\|}\angle V_2 - \angle$
$\delta_T(t)$	$\sum_{k=-\infty}^{\infty} \omega_0\delta(\omega - k\omega_0)$

Note that:

1. rect(t) = 1 for |t|<1/2 and 0 otherwise

2. sinc(t) = sin(t)/t

3. tri(t) = 1-|t| for |t|<1 and 0 otherwise.

4. u(t) is the step function (same as our q(t).)