HW 8 Solutions

Note Title 10/23/2007

1. Show that $v(t) = e^{j\omega t}$ is an eigen-signal with $H(\omega)$ as the corresponding eigen-value.

Proof: Using the convolution formula, the zero-state output when the input is $e^{j\omega t}$ is given below:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{j\omega(t-\tau)}\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{j\omega t} \cdot e^{-j\omega\tau}\, d\tau$$

$\because e^{j\omega t}$ does not depend on τ

$$= e^{j\omega t} \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= e^{j\omega t}\, H(\omega)$$

2. a) Using the table, we get the inverse fourier transform to be

$$h(t) = \mathcal{F}^{-1}\left[\mathrm{Rect}\left(\frac{\omega}{2\pi}\right)\right] = \frac{2\pi}{\pi}\,\mathrm{sinc}(2\pi t) = 2\,\mathrm{sinc}(t)$$

b) The phase indicates a negative delay of

$$t_0 = -\frac{\pi/2}{\pi} = -\frac{1}{2} \text{ from part a}$$

$$\Rightarrow \quad h(t) = 2\,\mathrm{sinc}[(t - t_0)] = 2\,\mathrm{sinc}\left[t + \frac{1}{2}\right]$$

c) The magnitude spectrum indicates

$$h_0(t) = \frac{1}{\pi} \cos(5\pi t)$$

The phase indicate a negative delay of

$t_0 = -\frac{\pi}{5\pi} = -\frac{1}{5} \Rightarrow$

$$h(t) = h_0(t - t_0) = \frac{1}{\pi} \cos[5\pi(t + \frac{1}{5})]$$

3 a) $y(t) = rect(t) * \cos(\pi t)$

$$\mathcal{F}[y(t)] = \mathcal{F}[rect(t) * \cos(\pi t)]$$

$$= \mathcal{F}[rect(t)] \cdot \mathcal{F}[\cos(\pi t)]$$

$$= sinc(\omega/2) \cdot [\pi \delta(\omega - \pi) + \pi \delta(\omega + \pi)]$$

$$= \pi sinc(\tfrac{\pi}{2}) \delta(\omega - \pi) + \pi sinc(-\tfrac{\pi}{2}) \delta(\omega + \pi)$$

$$= \cancel{\pi} \frac{1}{\cancel{\pi}/2} \delta(\omega - \pi) + \cancel{\pi} \frac{1}{\cancel{\pi}/2} \delta(\omega + \pi)$$

$$= \frac{1}{2} [\delta(\omega - \pi) + \delta(\omega + \pi]$$

$$y(t) = \frac{1}{2\pi} \cos(\omega t)$$

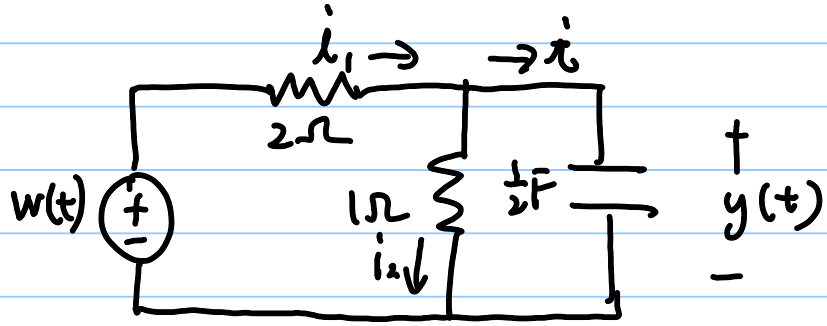
b) $\mathcal{F}[y(t)] = \text{sinc}^2(\omega/2)$

$$y(t) = \mathcal{F}^{-1}[\text{sinc}(\omega/2)] * \mathcal{F}^{-1}[\text{sinc}(\omega/2)]$$

$$= \text{rect}(t) * \text{rect}(t)$$

$$= \text{tri}(t)$$

4)



a)

$$\frac{1}{2}\frac{dy}{dt} = i = i_1 - i_2 = i_1 - y(t)$$

$$w(t) = 2i_1 + y(t) \Rightarrow i_1 = \frac{1}{2}[w(t) - y(t)]$$

$$\Rightarrow \quad \frac{1}{2}\frac{dy}{dt} = \frac{1}{2}[w(t) - y(t)] - y(t)$$

$$\frac{dy}{dt} + 3y(t) = w(t)$$

b) Apply FT onto the differential equations

$$j\omega Y(\omega) + 3Y(\omega) = W(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{W(\omega)} = \frac{1}{j\omega + 3}$$

c) The input $w(t) = e^{-3t} q(t) \Rightarrow W(\omega) = \dfrac{1}{3 + j\omega}$

$$Y(\omega) = H(\omega) W(\omega) = \frac{1}{(j\omega + 3)^2} \Rightarrow y(t) = t e^{-3t} q(t)$$

5) In order to reject 60 Hz and all harmonics, we need the spectrum of the impulse response to be

$$H(\omega) = 0 \quad \text{for}$$

$$\omega = 2\pi \cdot 60 \cdot n \qquad \text{for } n = \pm 1, \pm 2, \pm 3, \ldots$$

Given $\quad h(t) = A[q(t) - q(t - t_0)]$

$$H(\omega) = A\left[\pi\delta(\omega) + \frac{1}{j\omega} - e^{j\omega t_0}\pi\delta(\omega) - e^{j\omega t_0}\frac{1}{j\omega}\right]$$

$$= A\left[\cancel{\pi\delta(\omega)} - \cancel{\pi\delta(\omega)} + \frac{1}{j\omega}[1 - e^{j\omega t_0}]\right]$$

$$= \frac{A}{j\omega}[1 - e^{j\omega t_0}]$$

Since $\dfrac{A}{j\omega}$ is never zero, the only possibility is to have $\quad 1 - e^{j\omega t_0} = 0 \quad$ for $\omega = 2\pi \cdot 60 \cdot n$

Thus we have $e^{j\omega t_0} = 1$ for $\omega = 2\pi \cdot 60 \cdot n$

or $\omega t_0 = 2\pi n$ for $\omega = 2\pi \cdot 60 \cdot n$

or $\cancel{2\pi} \cdot 60 \cdot \cancel{n} t_0 = \cancel{2\pi} \cancel{n}$

$$t_0 = \frac{1}{60}$$