

University of Kentucky
Department of Electrical and Computer Engineering

EE421G: Signals and Systems I – Fall 2007

Problem Set 9

Issued: October 29, 2007

Due: November 5, 2007 (In class)

Reading Assignments:

Read Chapter 5.1 – 5.3 of Chen

Paper and Pencil Assignments:

1) Problem 5.1

Solution:

$x[n] \rightarrow \infty$ as $n \rightarrow \infty$, its spectrum is not defined. The sequence is not absolutely summable.

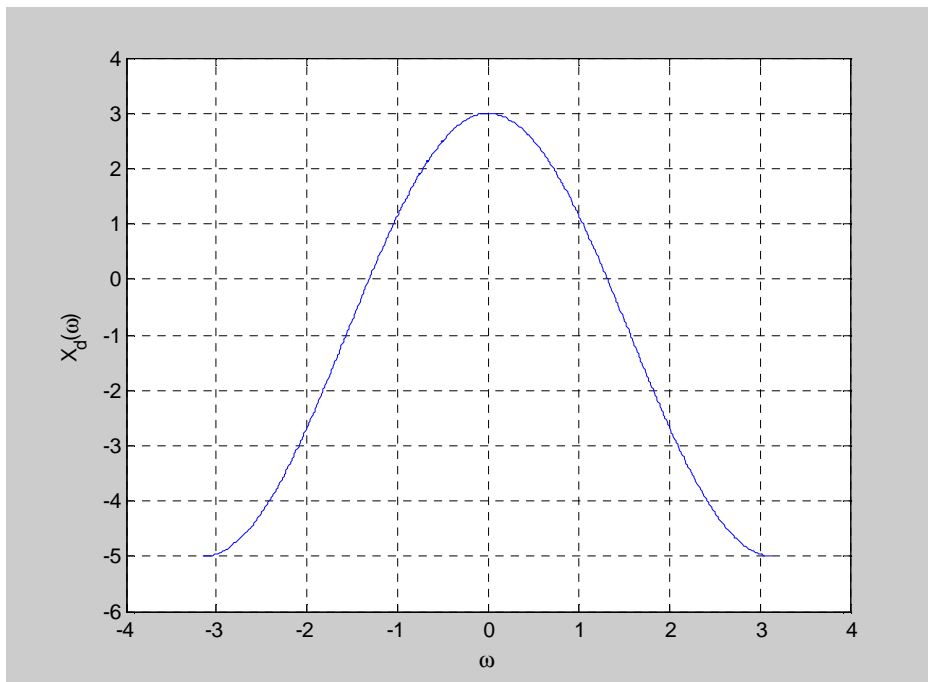
$$\left(\sum_{n=0}^{\infty} 2^n = \infty \right)$$

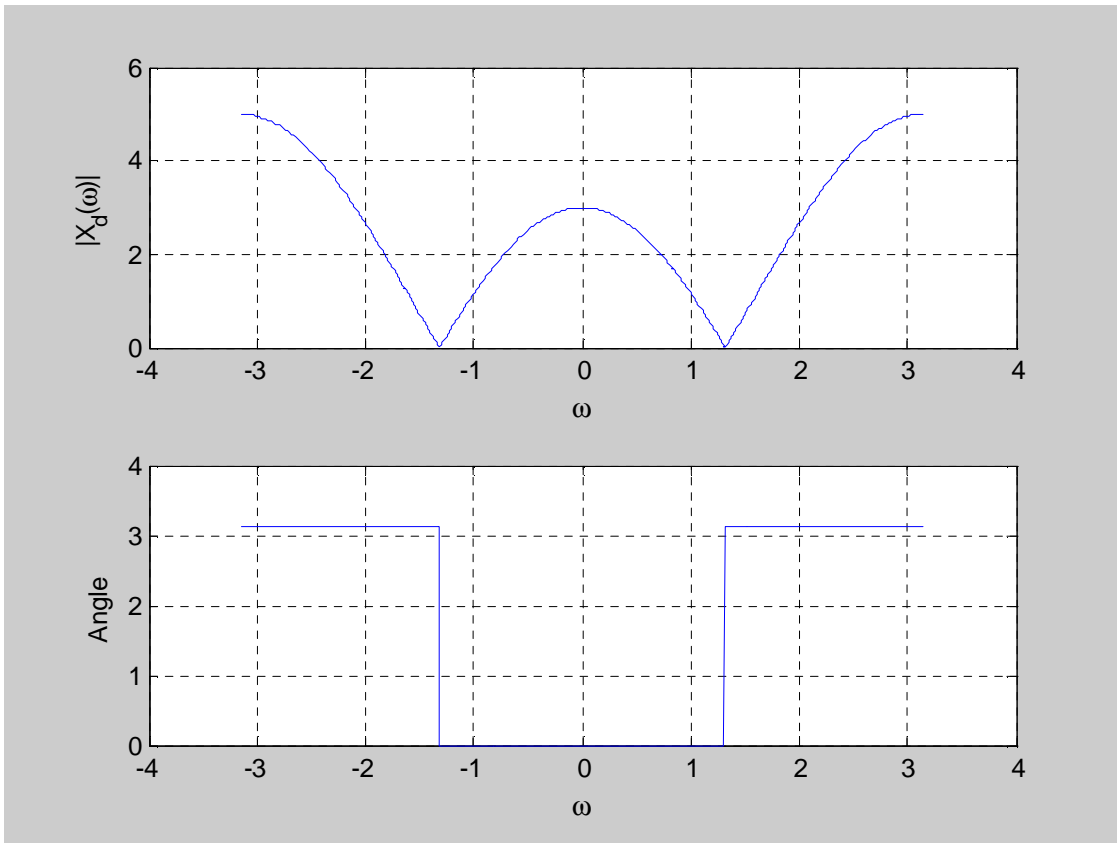
2) Problem 5.2

Solution:

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega T} = 2e^{j\omega} - 1e^{j0} + 2e^{-j\omega} = 4 \cos \omega - 1$$

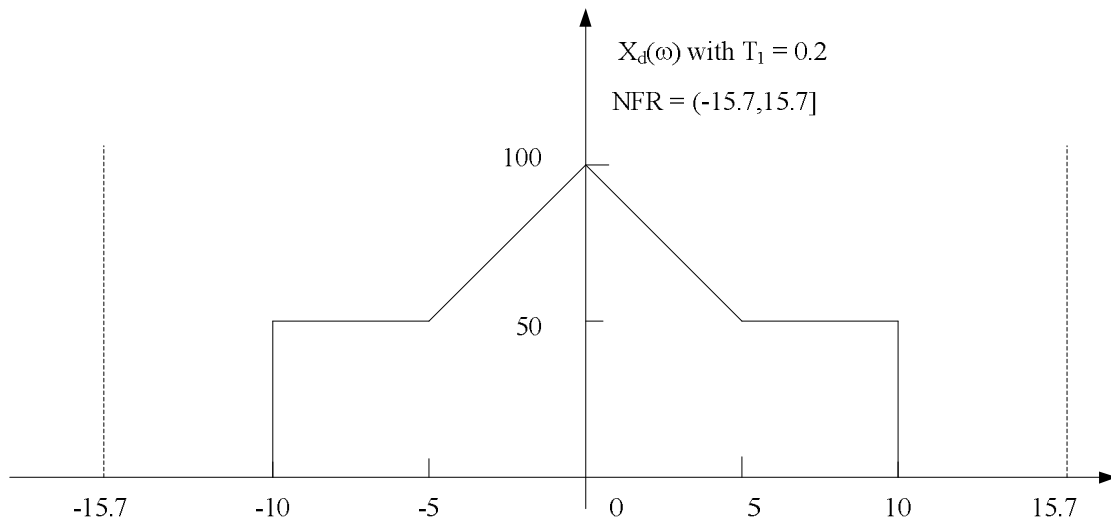
It is real valued, periodic with period 2π . Its NFR = $\left(-\frac{\pi}{T}, \frac{\pi}{T}\right] = (-\pi, \pi]$.

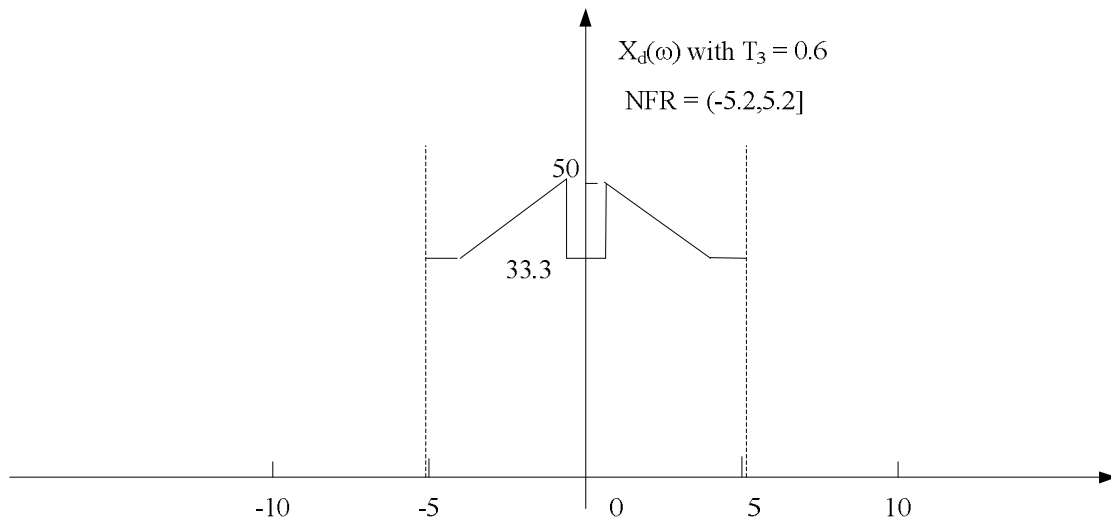
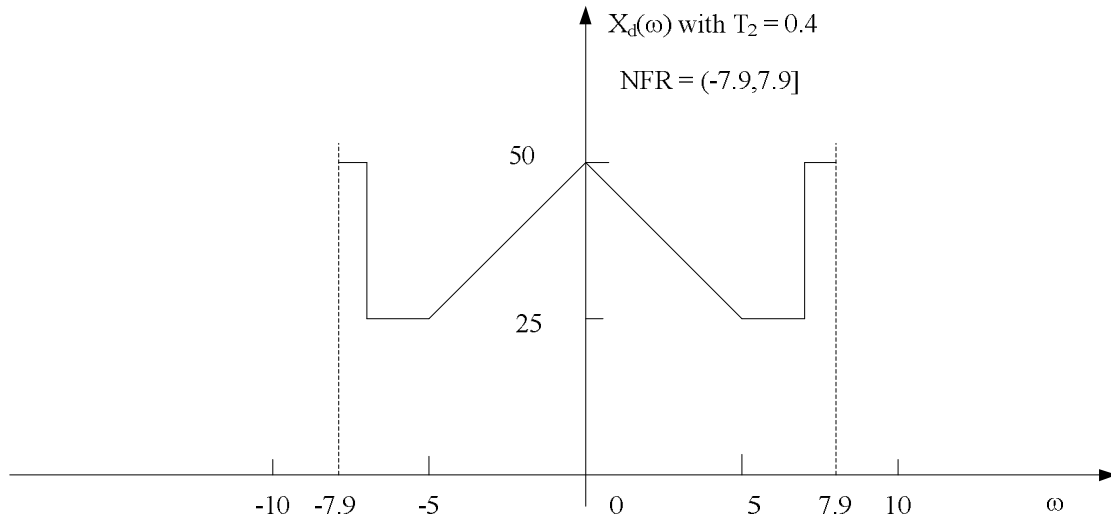




3) Problem 5.8

Solution:





If $T < \pi/W = \pi/10 = 0.314$, then there is no frequency aliasing and $TX_d(\omega)$ and $X(\omega)$ are the same inside the NFR. Thus $X(\omega)$ or $x(t)$ can be recovered from $X_d(\omega)$ or $x(nT)$. Note that $TX_d(\omega)$ and $X(\omega)$ are always different outside the NFR because $TX_d(\omega)$ can be extended periodically with period $2\pi/T$ outside the range and $X(\omega)$ is zero outside the range.

4) Problem 5.11

Solution:

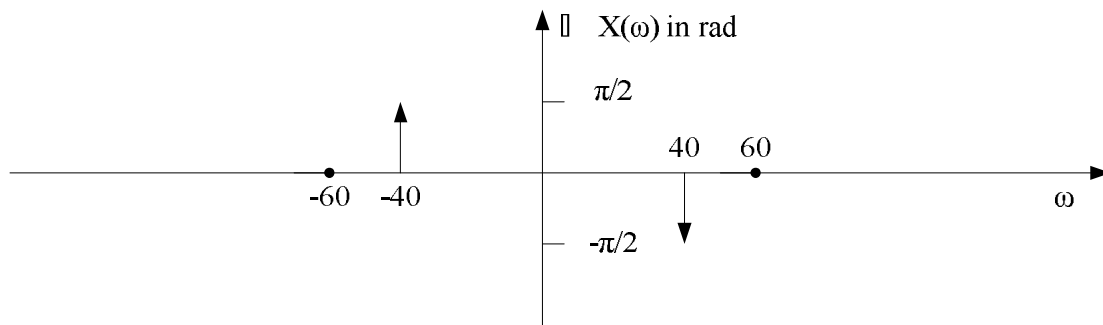
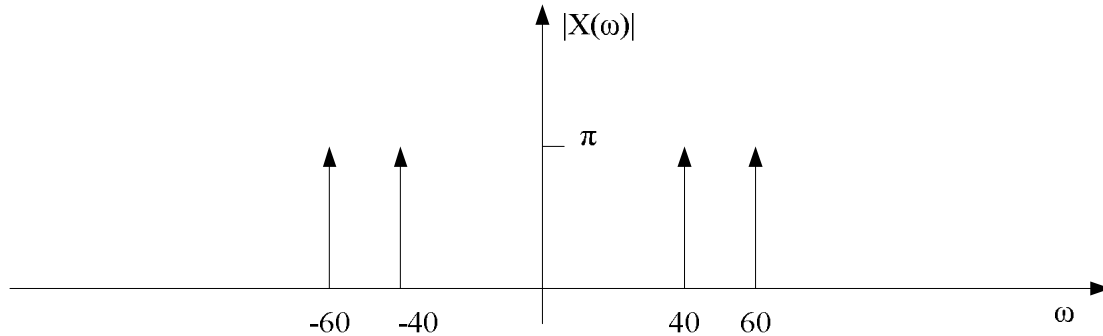
$$x(t) = \sin 40t + \cos 60t$$

Using (4.53) and (4.54), we have

$$X(\omega) = -j\pi\delta(\omega - 40) + j\pi\delta(\omega + 40)$$

$$+ \pi\delta(\omega - 60) + \pi\delta(\omega + 60)$$

It is complex-valued. We plot its magnitude and phase spectra.



$$x(nT) = \sin 40nT + \cos 60nT$$

$$T_1 = \pi/100 : NFR = (-100, 100]$$

$$X_d(\omega) = -j100\delta(\omega - 40) + j100\delta(\omega + 40)$$

$$+ 100\delta(\omega - 60) + 100\delta(\omega + 60)$$

There is no frequency aliasing and we have

$$X(\omega) = \begin{cases} X_d(\omega) & \text{for } \omega \text{ in } (-100, 100] \\ 0 & \text{for } |\omega| > 100 \end{cases}$$

$$T_2 = \pi/50 : NFR = (-50, 50]$$

$$x(nT_2) = \sin 40nT_2 + \cos 60nT_2$$

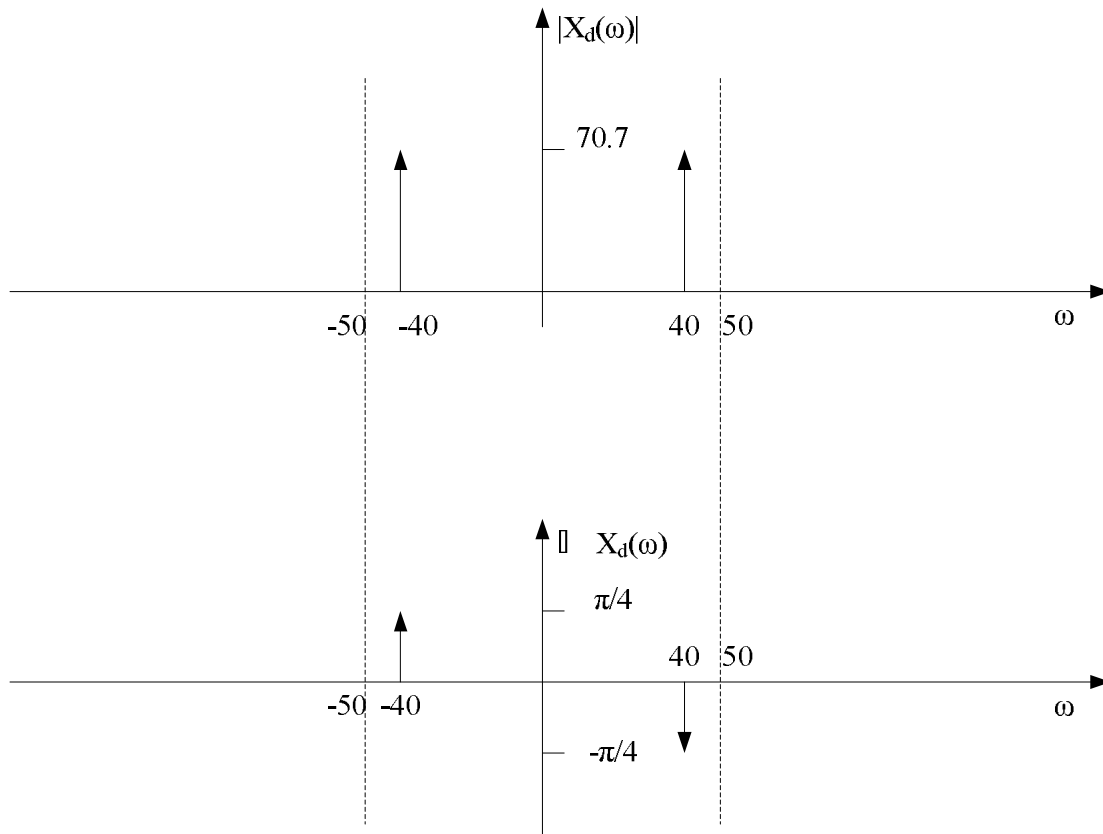
$$= \sin 40nT_2 + \cos(60 - 100)nT_2$$

$$= \sin 40nT_2 + \cos 40nT_2$$

Using (5.16) and (5.17), we have

$$\begin{aligned}
X_d(\omega) &= -j50\delta(\omega-40) + j50\delta(\omega+40) \\
&+ 50\delta(\omega-40) + 50\delta(\omega+40) \\
&= (50 - j50)\delta(\omega-40) + (50 + j50)\delta(\omega+40) \\
&= 70.7e^{-j\pi/4}\delta(\omega-40) + 70.7e^{j\pi/4}\delta(\omega+40)
\end{aligned}$$

There is frequency aliasing and we cannot obtain $X(\omega)$ from $X_d(\omega)$.



$$T_3 = \pi/30: NFR = (-30, 30]$$

$$x(nT_3) = \sin 40nT_3 + \cos 60nT_3$$

$$= \sin(40 - 60)nT_3 + \cos(60 - 60)nT_3$$

$$= -\sin 20nT_3 + 1$$

$$X_d(\omega) = j30\delta(\omega-20) - j30\delta(\omega+20) + 60\delta(\omega)$$

There is frequency aliasing.

5) Problem 5.12

Solution:

The spectrum of $e^{-0.05t}$, $t \geq 0$ is computed in (4.30) as $X(\omega) = 1/(j\omega + 0.05)$

It is not band limited. The spectrum of $x(nT) = e^{-0.05nT}$ is, as in (5.8),

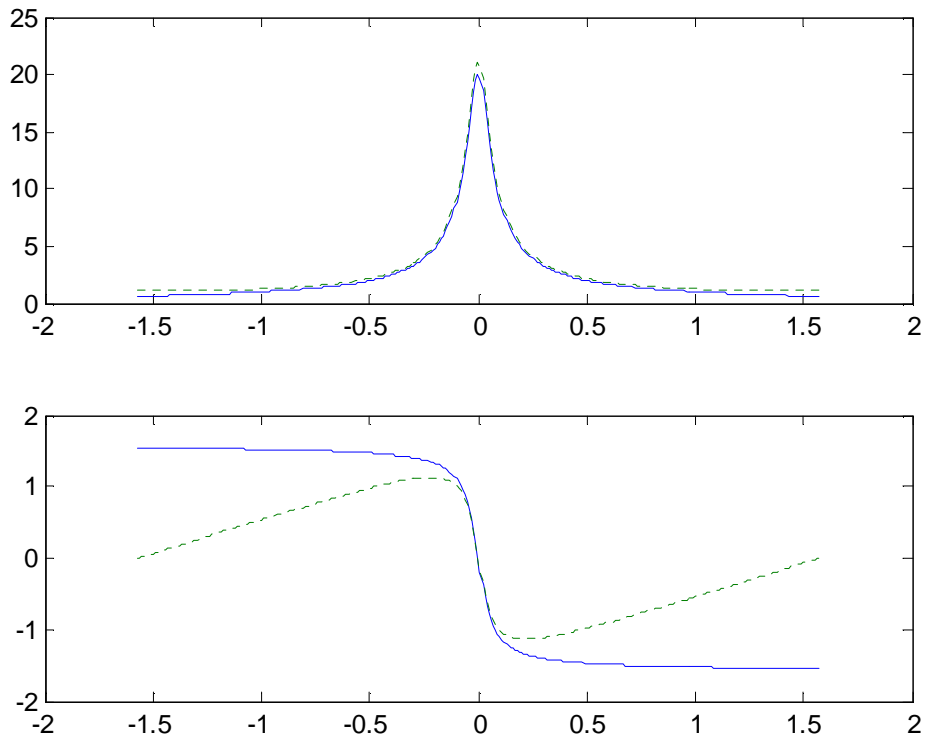
$$X_d(\omega) = \frac{1}{1 - e^{-0.05T} e^{-j\omega T}}$$

Typing in MATLAB

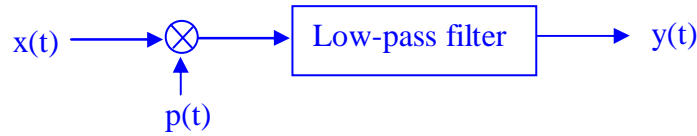
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T=2;
w=-pi/T:0.01:pi/T;
X=1./(j*w+0.05);
Xd=T./(1-exp(-0.05*T)*exp(-j*w*T));
subplot(2,1,1);
plot(w,abs(X),w,abs(Xd),'-');
subplot(2,1,2);
plot(w,angle(X),w,angle(Xd),'-');
```

yields in the top figure the magnitude spectra of $X(\omega)$ (solid line) and $TX_d(\omega)$ (dotted line)

in the NFR $(-\frac{\pi}{T}, \frac{\pi}{T}]$, and in the bottom figure their phase spectra. There is appreciable frequency aliasing.



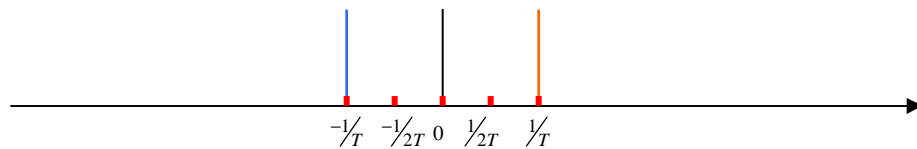
- 6) As described in the question, we wish to reconstruct $y(t)$ based on the following block diagram:



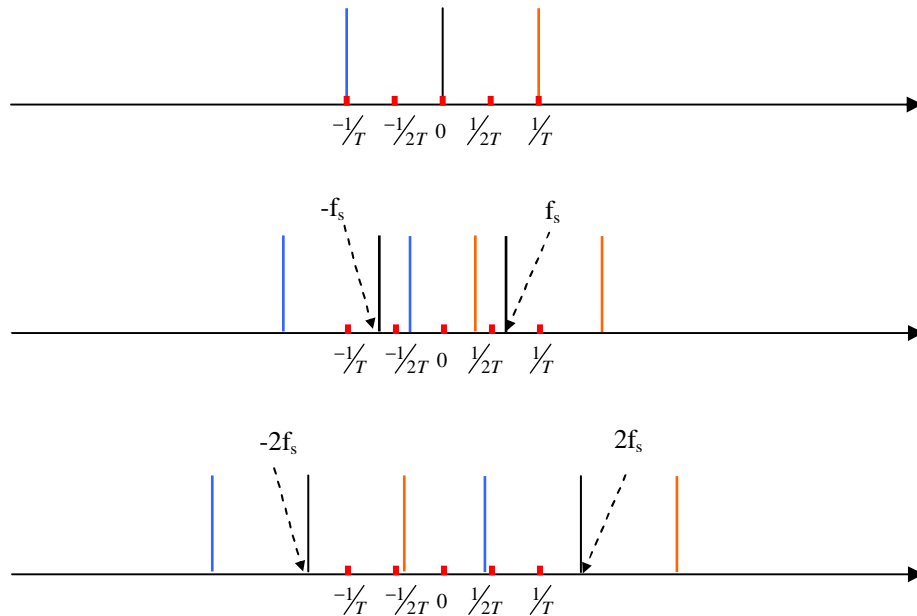
To analyze what the oscilloscope is doing, we start with the Fourier spectrum of the signal $A+B\cos[(2\pi/T)t+\theta]$ which is

$$X(f) = A\delta(f) + \frac{Be^{j\theta}}{2}\delta(f - \frac{1}{T}) + \frac{Be^{-j\theta}}{2}\delta(f + \frac{1}{T})$$

Pictorially, the spectrum consists of three delta functions with different values:



Let $f_s=1/(T+\Delta)<1/T$. This is substantially less than the Nyquist rate which is $2/T$. The following shows the original spectrum and the first two periodic extensions to both the negative and positive directions. For clarity, they are shown in separate graphs even though they are all present simultaneously. We do not need to consider the third periodic extensions and beyond as their frequencies are too high to affect the low-pass interpolation filter.



It is clear from the above drawing that in order to have the low-pass region resembled the original spectrum, the red bar (second from the right) in the second graph must be closer to the origin than the blue bar (first from the right) in the third graph.

$$\text{Location of the red bar} = -f_s + 1/T = -1/(T+\Delta) + 1/T$$

$$\text{Location of the blue bar} = 2f_s - 1/T = 2/(T+\Delta) - 1/T$$

The condition is thus: $-1/(T+\Delta) + 1/T < 2/(T+\Delta) - 1/T$

$$\Rightarrow 3/(T+\Delta) > 2/T$$

$$\Rightarrow \Delta < T/2$$

If this condition is satisfied, then the cutoff frequency can be set to the location of the rightmost position of the red bar in the second graph, i.e. $-1/(T+T/2) + 1/T = 1/(3T)$