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Midterm 1 (Fall 2007) of EE421G:

1. This midterm consists of four single-sided pages. The last page is an extra worksheet. You can tear out any page but make sure all the pages you turn in have your name on them.
2. There are three problems in this exam. You have 50 minutes to finish this exam.
3. You are allowed to use one double-sided page of cheat sheet.

Good luck!

1. (Outcome 2, 34 points) Consider the signal $f(t) = \cos\left(3t + \frac{\pi}{2}\right) + \sin\left(5t + \frac{\pi}{2}\right)$.

a. Is the signal $f(t)$ periodic? If yes, what is its period?

The period of the cosine function is $2\pi/3$ and that of the sine function is $2\pi/5$. If their sum is periodic with period T , T must be an integral multiples of the period of the constituent components, or $T=m2\pi/3$ and $T=n2\pi/5$. The smallest integers that satisfy these equations are $m=3$ and $n=5$. Since such multipliers exist, $f(t)$ is periodic with period $T=2\pi$.

b. Write $f(t)$ in terms of complex exponentials.

$$\begin{aligned} f(t) &= \cos(3t + \pi/2) + \sin(5t + \pi/2) \\ &= 0.5\{\exp[j(3t+\pi/2)] + \exp[-j(3t+\pi/2)]\} - 0.5j\{\exp[j(5t+\pi/2)] - \exp[-j(5t+\pi/2)]\} \\ &= 0.5[j\exp(j3t) - j\exp(-j3t)] - 0.5j[j\exp(j5t) + j\exp(-j5t)] \\ &= 0.5j\exp(j3t) - 0.5j\exp(-j3t) + 0.5\exp(j5t) + 0.5\exp(-j5t) \end{aligned}$$

c. Plot the magnitude and phase of the frequency spectrum based on your answer in b.

There are only four non-zero frequency components at $\omega = -5, -3, 3$ and 5 rad/s.

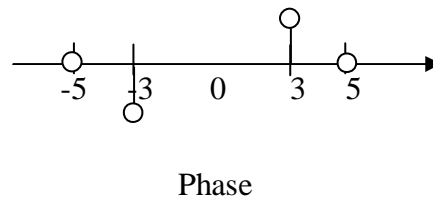
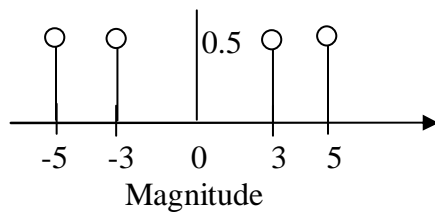
The frequency spectrum at each frequency is simply the coefficient to the complex exponential at that frequency, i.e.

Frequency spectrum at $\omega = -5$ rad/s is 0.5 (or magnitude=0.5 and phase=0).

Frequency spectrum at $\omega = -3$ rad/s is $-0.5j$ (or magnitude=0.5 and phase= $-\pi/2$).

Frequency spectrum at $\omega = 3$ rad/s is $0.5j$ (or magnitude=0.5 and phase= $\pi/2$).

Frequency spectrum at $\omega = 5$ rad/s is 0.5 (or magnitude=0.5 and phase= 0).



d. Now define a discrete-time signal $g[n] = f(2n)$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$. Is the signal $g[n]$ periodic? If yes, what is its period?

$$g[n] = f(2n) = \cos(6n + \pi/2) + \sin(10n + \pi/2).$$

For $g[n]$ to be periodic with period N , we need $6N=2\pi m$ and $10N=2\pi n$ for integer m and n . However, as π is an irrational number, no integer N can satisfy these constraints and thus $g[n]$ is not periodic.

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2. (Outcome 1, 36 points) Consider the following three systems (OUTCOME 1):

SYSTEM A $y(t) = u(t+2)\sin(\omega t + 2)$, where $\omega \neq 0$

SYSTEM B $y[n] = \left(-\frac{1}{2}\right)^n (u[n]+1)$

SYSTEM C $y[n] = \sum_{k=1}^n (u[k+1]^2 - u[k])$

where $u[n]$ and $y[n]$ are the input and output of each system.

Circle YES or NO for each of the following questions for each of these three systems.

	SYSTEM A		SYSTEM B		SYSTEM C	
Is the system memory-less?	YES	NO	YES	NO	YES	NO
Is the system causal?	YES	NO	YES	NO	YES	NO
Is the system time invariant?	YES	NO	YES	NO	YES	NO
Is the system linear?	YES	NO	YES	NO	YES	NO

System A is not memoryless because the output at t depends on input at $t+2$. Since it depends on future input, it is clearly not causal. If we change the input from $u(t)$ to $u(t+T)$, the output will be $u(t+T+2)\sin(\omega t+2)$, which is not the same as $y(t+T)=u(t+T+2)\sin(\omega t+\omega T+2)$. So it is not time invariant. If we use the input $au_1(t)+bu_2(t)$, the output is $[au_1(t+2)+bu_2(t+2)]\sin(\omega t+2) = au_1(t)\sin(\omega t+2) + bu_2(t)\sin(\omega t+2) = ay_1(t)+by_2(t)$. Thus the system is linear.

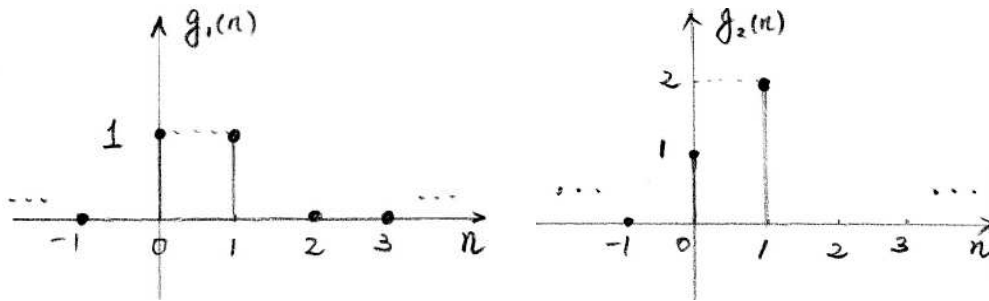
System B is memoryless because the output at n depends only on input at n . Since it is memoryless, it must be causal. If we change the input from $u[n]$ to $u[n+N]$, the output is $(-1/2)^n(u[n+N]+1)$ which is not the same as $y[n+N]=(-1/2)^{n+N}(u[n+N]+1)$. Thus it is not time invariant. If we use the input $au_1[n]+bu_2[n]$, the output is $(-1/2)^n(au_1[n]+bu_2[n]+1)$, which is not the same as $ay_1[n]+by_2[n] = a(-1/2)^n(u_1[n]+1)+b(-1/2)^n(u_2[n]+1) = (-1/2)^n (au_1[n]+bu_2[n]+a+b)$. Thus, it is not linear.

System C is not memoryless because the output at n depends on all input from 1 to $n+1$. It is not causal because it depends on the future input. If we change the input from $u[n]$ to $u[n+N]$, the output is $\sum_{k=1}^{n+N} (u[k+1]^2 - u[k]^2) = y(n+N)$ and thus the system is time invariant. If we use the input $au_1[n]+bu_2[n]$, the output is $\sum_{k=1}^{n+N} [(au_1[k+1]+bu_2[k+1])^2 - (au_1[k]+bu_2[k])^2]$ which is certainly not $ay_1[n]+by_2[n]$. The system is not linear.

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(Outcome 6 & 7, 30 points)

- a. Calculate the convolution $g_1[n] * g_2[n]$ of the two discrete-time signals shown below.

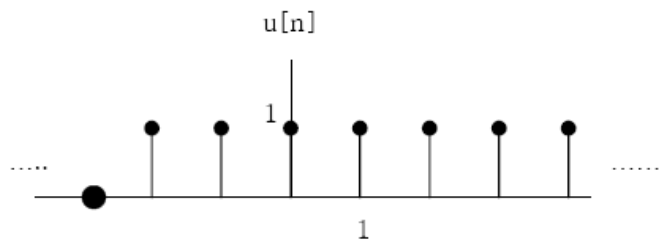


A simple graphical computation will obtain
 $y[0] = 1, y[1] = 3, y[2] = 2$

- b. Consider an LTI system with the input and output related by $y[n] = u[n+2] + u[n]$
 i) Find the system impulse response $h[n]$.

$h[n] = \delta[n+2] + \delta[n]$

- ii) Determine the output $y[n]$ for the input shown below.



$u[n] = q[n+2]$ so $y[n] = q[n+4] + q[n+2]$ or

$y[n] = 1$ for $n = -4, -3$
 2 for $n > -3$
 0 otherwise

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