Note Title 11/29/2007

Midterm 3

1. Discrete time Fourier Transform DTFT

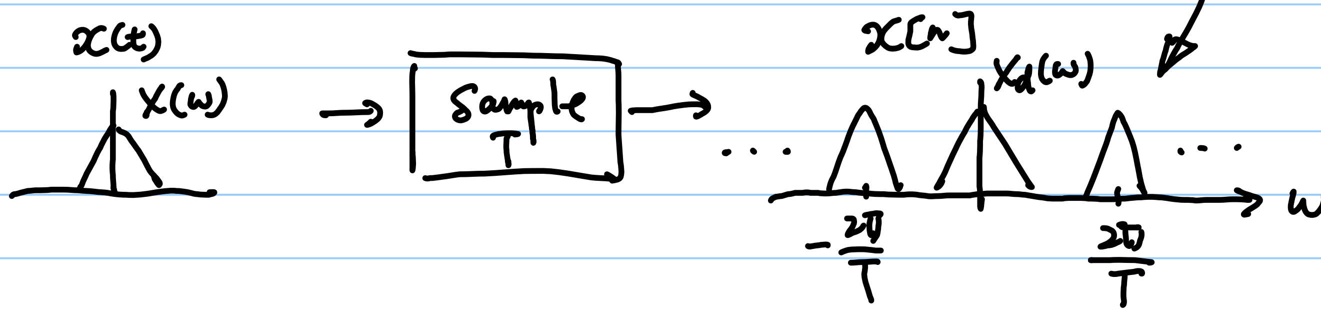
should know $x[n] \longrightarrow X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$|X_d(\omega)|$, $\angle X_d(\omega)$

Won't be on the test $X_d(\omega) \xrightarrow{T=1} x[n] = \frac{1}{2\pi}\int_{-\frac{1}{\pi}}^{\frac{1}{\pi}} X_d(\omega) e^{j\omega n}\, d\omega$

2 Properties of DTFT
- periodic

Sampling



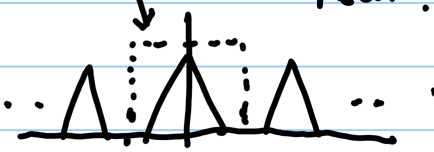
Linearity $\mathcal{F}\{x[n] + y[n]\} = X_d(\omega) + Y_d(\omega)$

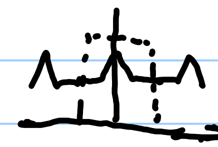
Eigensignal $e^{j\omega_0 n} \rightarrow \boxed{H_d(\omega)} \rightarrow H_d(\omega_0) e^{j\omega_0 n}$

Convolution $\mathcal{F}\{x[n] * y[n]\} = X_d(\omega)\, Y_d(\omega)$

3. Nyquist Theorem **

Ideal LPF ⇒ perfect recon.

Sample above the Nyquist Rate ⇒ 

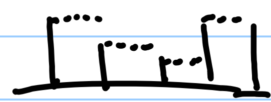
Sample below the Nyquist Rate ⇒ Aliasing 

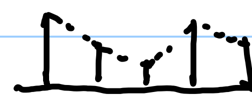
4. Practical D/A

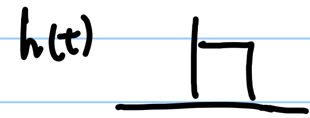
$$x[n] \longrightarrow \boxed{h(t)} \longrightarrow x(t)$$

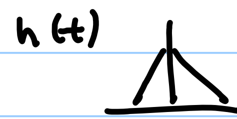
Ideal LPF is not practical. Why?

Practical : Sample-and-Hold , Linear Interpolation



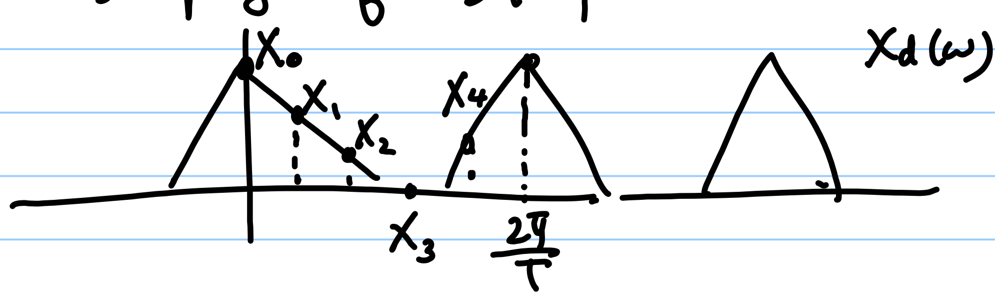






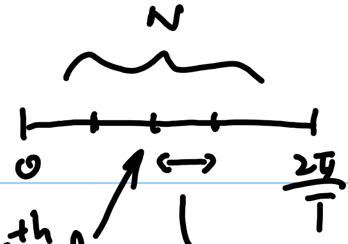
5. Discrete Fourier Transform (DFT)

- sampling of DTFT



X_0, X_1, X_2, X_3, X_4 — 5-pt DFT

* Evaluate $\Rightarrow$ $$X_k = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\frac{2\pi nk}{N}}$$



k^{th} freq

$\omega = \frac{2\pi}{TN}\cdot k$ $\quad \frac{2\pi}{TN}$

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega nT}$$

** If $x[n]$ has N or fewer samples, DFT is invertible

* Evaluate $\Rightarrow$ $$x[n] = \sum_{k=0}^{N-1} X_k\, e^{j\frac{2\pi nk}{N}} \qquad n = 0, 1, 2, \ldots, N-1$$

Ex. $x[n] = [1 \quad 2 \quad 3]$

3-pt

$$X_0 = x[0]\, e^{j\frac{2\pi 0\cdot 0}{3}} + x[1]\, e^{j\frac{2\pi 1\cdot 0}{3}} + x[2]\, e^{j\frac{2\pi 2\cdot 0}{3}} = 6$$

$$X_1 = x[0]\, e^{j\frac{2\pi 0\cdot 1}{3}} + x[1]\, e^{j\frac{2\pi 1\cdot 1}{3}} + x[2]\, e^{j\frac{2\pi 2\cdot 1}{3}} = \ldots$$

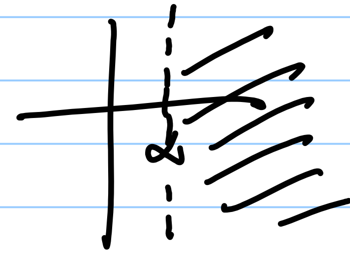
$$X_2 = x[0]\, e^{j\frac{2\pi 0\cdot 2}{3}} + x[1]\, e^{j\frac{2\pi 1\cdot 2}{3}} + x[2]\, e^{j\frac{2\pi 2\cdot 2}{3}}$$

6. Laplace Transform

$$X(s) = \mathcal{L}[x(t)] = \int_0^{\infty} x(t)\, e^{-st}\, dt$$

- wider application than FT

$\mathcal{L}[\delta(t)] = 1$ $\qquad$ $\mathcal{L}[e^{-\alpha t}] = \frac{1}{s+\alpha}$ $\quad$ ...

+ ROC  

- identify all the poles (values at which the Laplace transform become infinity)

ex $\frac{1}{s+\alpha}$ has a pole at $-\alpha$

ROC : Right of the right-most pole.

7. LT of ordinary differential Equation

$$a\frac{dy}{dt} + b\,y(t) = c\,u(t)$$

LT

initial value $y(t)\big|_{t=0^-}$

$$\to a\,sY(s) - y(0^-) + b\,Y(s) = c\,U(s)$$

$$\updownarrow \quad Y(s) = \underbrace{\frac{c\,U(s)}{as+b}}_{Y_{zs}(s)} + \underbrace{\frac{y(0^-)}{as+b}}_{Y_{zI}(s)}$$

transfer function $H(s)$

$$Y_{zs}(s) = \frac{c}{as+b} \cdot U(s) \triangleq H(s) \cdot U(s)$$

LT of the impulse response

$$\overset{\mathcal{L}^{-1}}{\Rightarrow} \quad y_{zs}(t) = h(t) * u(t)$$

8. Complex Impedance (Inductor, capacitor, resistor)

a, Inverse LT (5-step process).