Note Title 10/25/2007

① Show

$$u(t) = e^{j\omega t} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \underline{\underline{H(\omega)\cdot e^{j\omega t}}}$$

$$y(t) = \int h(\tau)\, u(t-\tau)\, d\tau$$

$$= \int h(\tau)\, e^{j\omega(t-\tau)}\, d\tau$$

$$\Big\downarrow \text{ easy}$$

$$= \int h(\tau)\, e^{j\omega\tau} d\tau \cdot e^{j\omega t}$$

Def of FT to get an integral

$$= H(\omega)\cdot e^{j\omega t}$$

(DONE)

② Table 1, 2

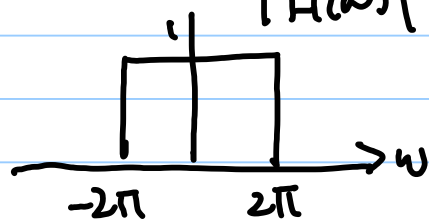
$$\mathrm{rect}(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

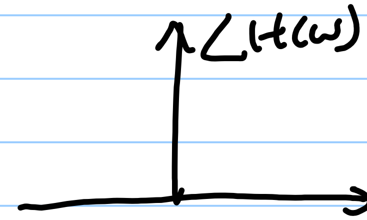
$$\mathrm{tri}(t) = \begin{cases} 1-|t| & |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$u(t)$ – step function ($q(t)$ in my lecture)

Inverse FT

a)





$\searrow \text{rect}\left(\frac{\omega}{4\pi}\right)$

rect(t) :

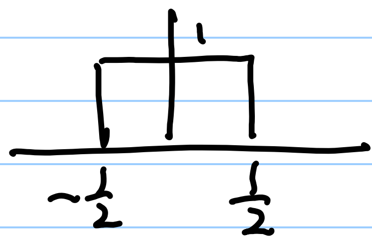


Table 2

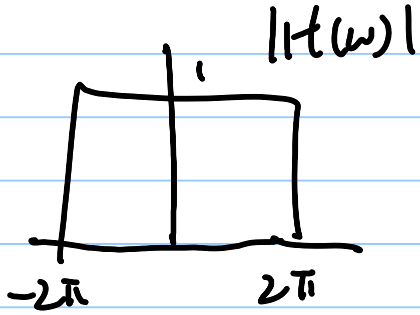
$\beta = 2\pi$

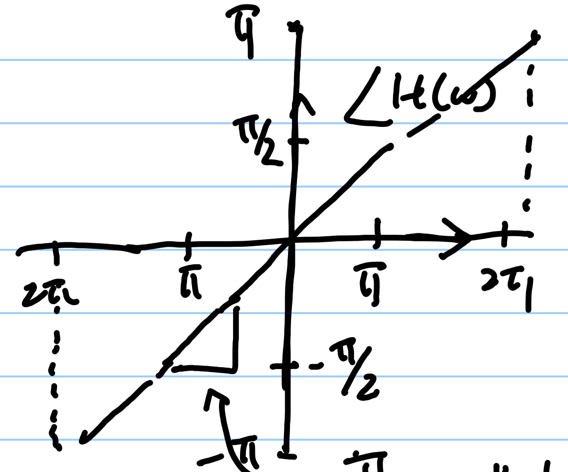
Table 2

Time	FT
⋮	⋮
$\frac{\beta}{\pi}\text{sinc}(\beta t)$	$\text{rect}\left(\frac{\omega}{2\beta}\right)$
⋮	⋮

$$h(t) = \frac{2\pi}{\pi}\text{sinc}(2\pi t) = 2\,\text{sinc}(2\pi t)$$

b)





$\frac{\pi}{2\pi} = \cancel{4}\ \frac{1}{2}$

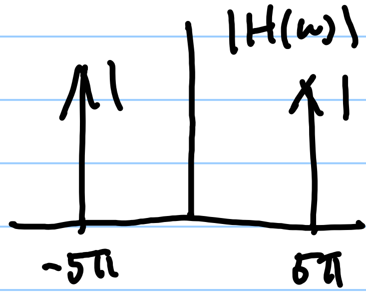
$$\angle H(\omega) = \frac{1}{2}\omega$$

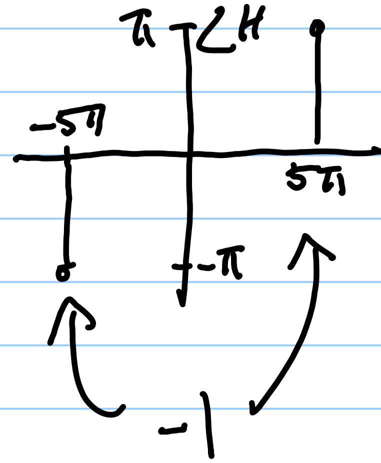
$$H(\omega) = |H(\omega)| \cdot e^{j\frac{1}{2}\omega} = \underbrace{|H(\omega)|\,e^{j0}}_{} \cdot e^{j\frac{1}{2}\omega}$$

$\boxed{\text{part a}} \rightsquigarrow H_a(\omega)$

Table 1	Time	FT
	⋮	⋮
	$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
	⋮	⋮

c)



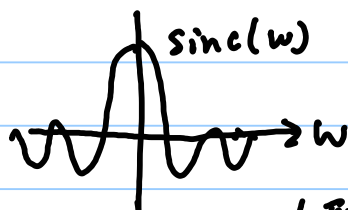


Hint: Table 2

3) Simplify $y(t)$ so that the is no convolution

a) $y(t) = \text{rect}(t) * \cos(\pi t)$

$\xrightarrow{\mathcal{F}}$



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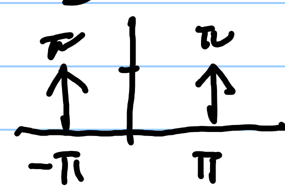
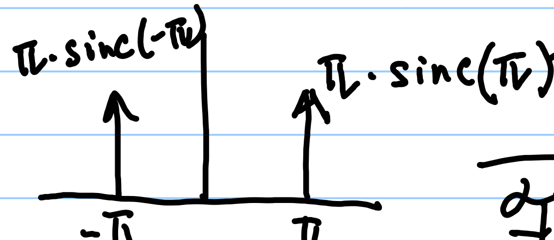


$$A = \int f(\omega)\,\delta(\omega-\pi)\,d\omega$$

$= f(\pi)$

– sifting

$=$



numerically

$\xrightarrow{\mathcal{F}^{-1}}$

b) $\mathcal{F}[y(t)] = \text{sinc}^2(\omega)$

– easy.

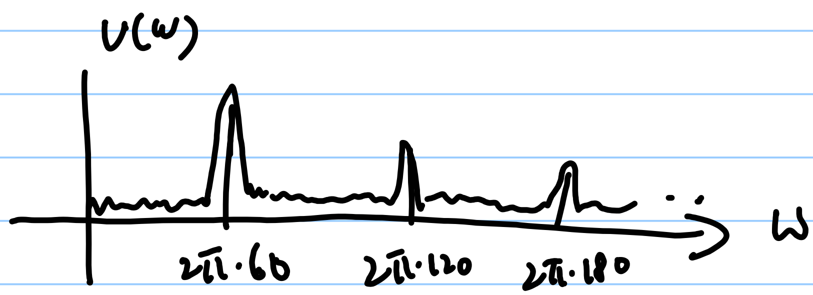
4) See example in class

5)

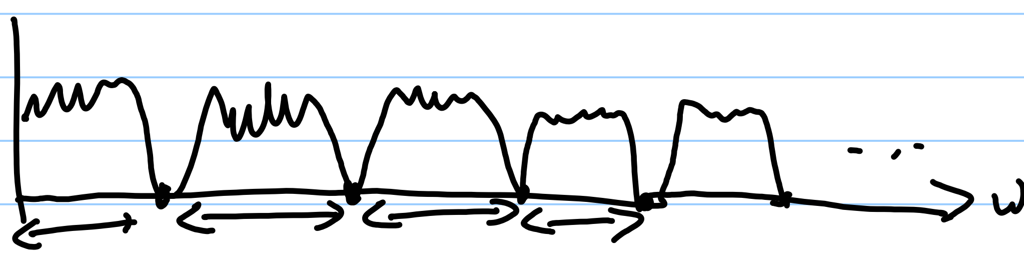
$$U(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega)$$

$$Y(\omega) = U(\omega)H(\omega)$$

$U(\omega)$ is noise : power supply 60Hz and its harmonics



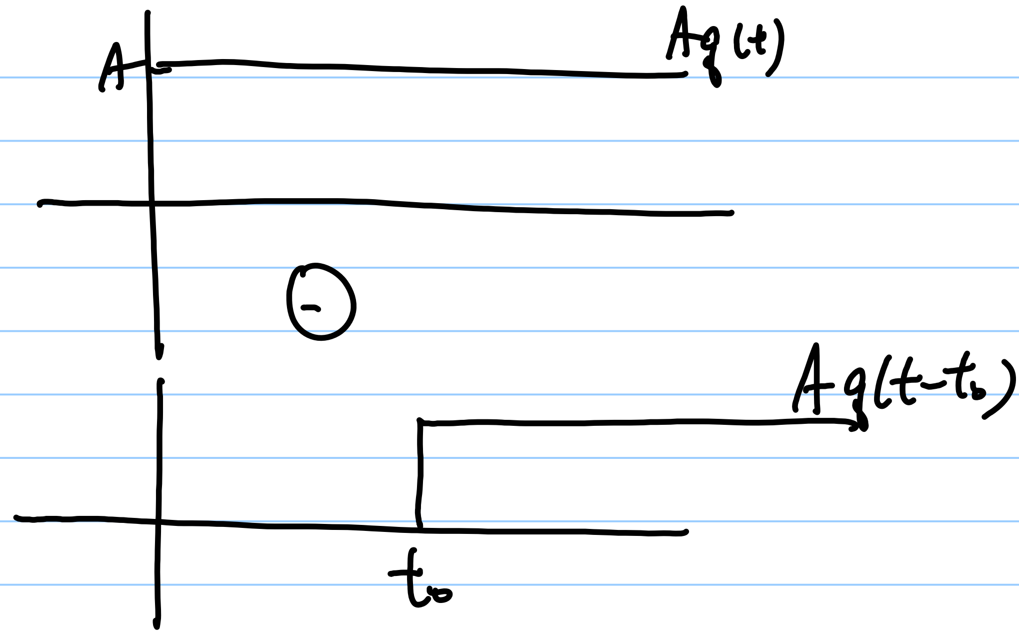
To design a $H(\omega)$: Comb filter



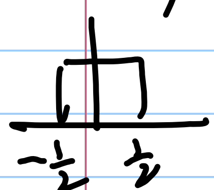
$$h(t) = A\left(q(t) - q(t-t_0)\right)$$

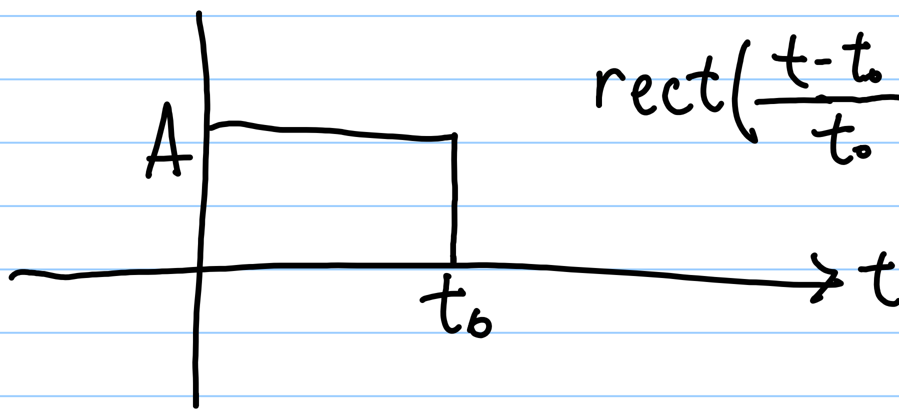
Question : what is t_0 ?

$h(t)$



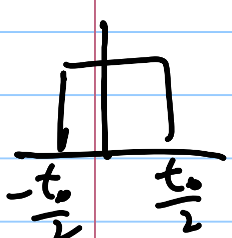
$\mathrm{rect}(t)$ =





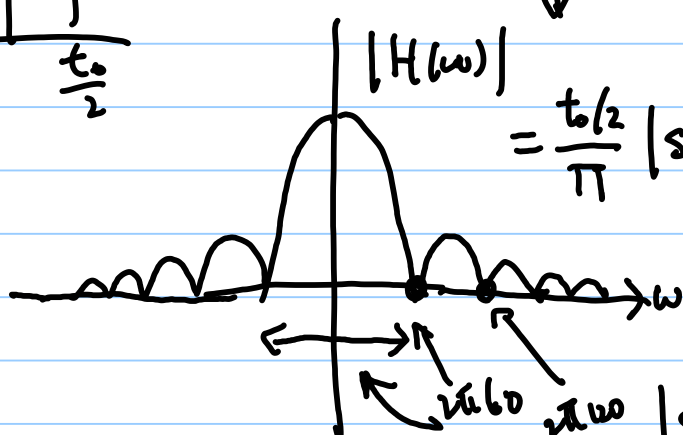
$\mathrm{rect}\left(\frac{t-t_0}{t_0}\right)$

$\mathrm{rect}\left(\frac{t}{t_0}\right)$



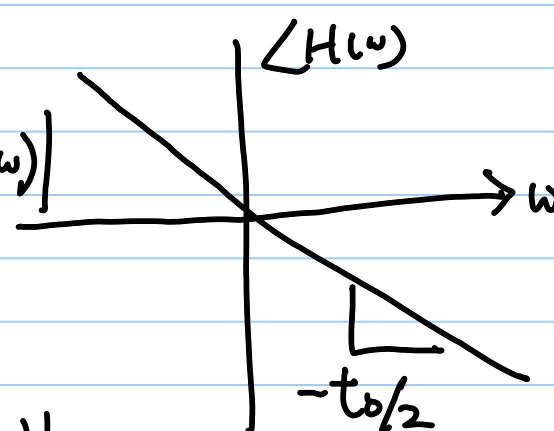
$\downarrow \mathcal{F}$

$|H(\omega)|$



$= \frac{t_0/2}{\pi} |\mathrm{sinc}(t_0 \omega)|$

$\angle H(\omega)$



$$\left|\frac{\sin(t_0 \omega)}{t_0 \omega}\right|$$

zero crossings

$\sin(0, \pi, 2\pi, 3\pi, \ldots) = 0$