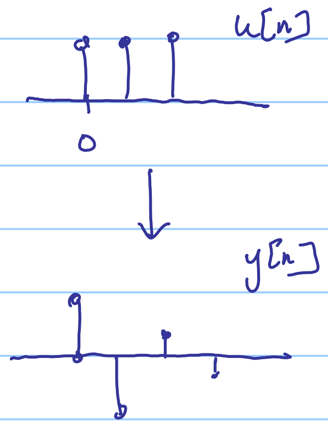
Lecture 12

Note Title 9/19/2007

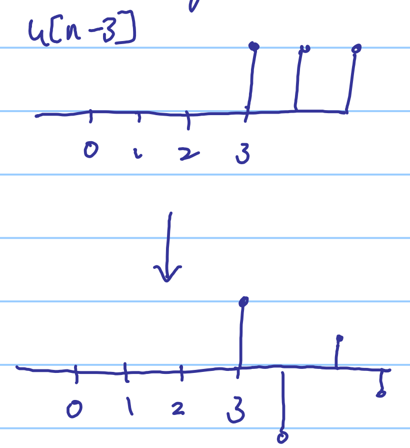
Outline
- properties of DT systems
- convolution to compute LTI DT system
- compute graphically
- IIR, FIR $\Rightarrow$ difference equation

DT system properties

- Time Invariance "Does not care when you start"



Delay by
3 time steps



If a system
$$\left.\begin{matrix} u[n], \; n \geq n_0 \\ x[n_0] = x_0 \end{matrix}\right\} \rightarrow y[n] \quad n \geq n_0$$

a system is time invariant if the following is true

$$\left.\begin{matrix} u[n-m], \quad n \geq n_0 + m \\ x[n_0 + m] = x_0 \end{matrix}\right\} \rightarrow y[n-m], \quad n \geq n_0 + m$$

- Linearity : Additivity + Homogenity

① Additivity if

$$\left.\begin{array}{ll} u_1[n] & n \geq n_0 \\ x_1[n_0] & \end{array}\right\} \longrightarrow y_1[n],\ n \geq n_0$$

$$\left.\begin{array}{ll} u_2[n] & n \geq n_0 \\ x_2[n_0] & \end{array}\right\} \longrightarrow y_2[n],\ n \geq n_0$$

$$\Rightarrow \quad \left.\begin{array}{ll} u_1[n] + u_2[n], & n \geq n_0 \\ x_1[n_0] + x_2[n_0] & \end{array}\right\} \rightarrow y_1[n] + y_2[n], \quad n \geq n_0$$

If a system is additive, you can always break down the input and state

$$u[n], \ x[n_0] = x_0 \longrightarrow y[n]$$

$$\begin{array}{lll} u_1[n] = u[n], \ n \geq 0 & \quad & u_2[n] = 0 \\ x_1[n_0] = 0 & & x_2[n_0] = x_0 \\ \downarrow & & \downarrow \\ y_{ZS}[n] & & y_{ZI}[n] \end{array}$$

By additivity $\quad y[n] = y_{ZS}[n] + y_{ZI}[n]$

② Homogenity : if

$$\left.\begin{array}{l} x[n_0] \\ u[n], \ n \geq n_0 \end{array}\right\} \longrightarrow y[n], \ n \geq n_0$$

$$\Rightarrow \quad \text{for } \forall \alpha \quad \left.\begin{array}{l} \alpha\, x[n_0] \\ \alpha\, u[n], \ n \geq n_0 \end{array}\right\} \longrightarrow \alpha\, y[n], \ n \geq n_0$$

Generality of linear system

① Even for non-linear system, if the input is restricted to a small range (small-signal approximation) the system is linear.

② $y(t) = \mathcal{U}[u(t)]$ is non-linear, it might

non-linear systems

be possible to do $y(t) = \mathcal{L}[f(u(t))]$

Linear

non-linear but not time varying "static"

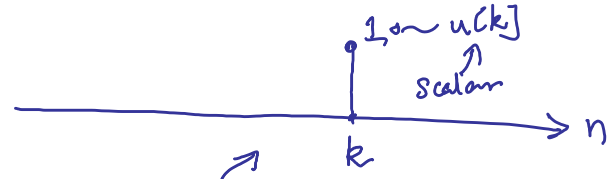
Linear and Time Invariant (LTI) systems

** The zero-state response $y_{zs}[n]$ of a LTI system
** can be computed using convolution

Sifting Properties

$$u[n] = \sum_{k=-\infty}^{\infty} u[k]\,\delta[n-k]$$

$$y[n] = \mathcal{L}[u[n]] = \mathcal{L}\left[\sum_{k=-\infty}^{\infty} u[k]\,\delta[n-k]\right]$$



By additivity $\quad y[n] = \sum_{k=-\infty}^{\infty} \mathcal{L}\left[u[k]\, \delta[n-k] \right]$

$$u[n] \longrightarrow \boxed{\mathcal{L}} \longrightarrow y[n]$$

time index

$\because$ homogenity $\quad = \sum_{k=-\infty}^{\infty} u[k]\, \mathcal{L}\left[\delta[n-k]\right] \qquad (\triangle)$

if $h[n] \triangleq \mathcal{L}[\delta[n]]$, $\because$ time invariant $\Rightarrow h[n-k] = \mathcal{L}[\delta[n-k]]$

Sub back to $(\triangle)$ $\quad y[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$ "convolution"

$\qquad\qquad = u[n] * h[n]$

$h[n]$ = impulse response.

Significance : instead of using our black box for all possible input, all we need is to use it once to get $h[n]$

And then use the convolution to compute $y[n]$ for any input $u[n]$

Properties of convolution

① Commutative i.e $\quad u[n] * h[n] = h[n] * u[n]$

Why? $\quad y[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k] = u[n] * h[n]$

Let $\bar{k} = n-k$

$$y[n] = \sum_{\bar{k}=+\infty}^{-\infty} u[n-\bar{k}]\, h[\bar{k}]$$

$$= \sum_{\bar{k}=-\infty}^{\infty} u[n-\bar{k}]\, h[\bar{k}]$$

$$= \sum_{k=-\infty}^{\infty} u[n-k]\, h[k] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$$= h[n] * u[n]$$

② Causality :

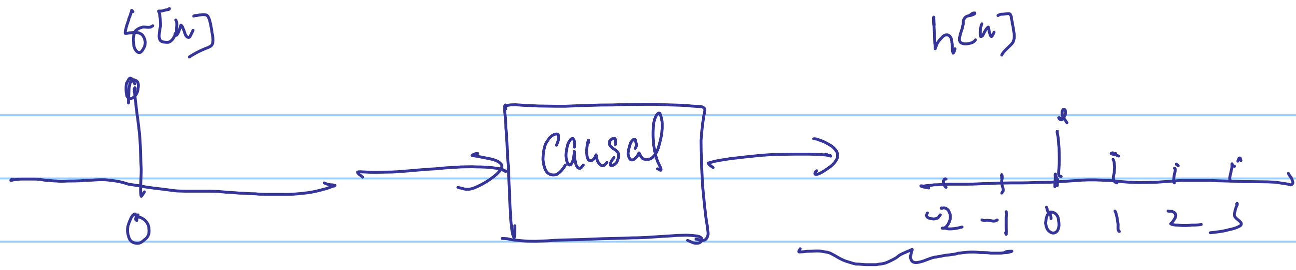
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$$= \dots + \underset{\uparrow\ \text{future}}{h[-2]u[n+2]} + \underset{\uparrow\ \text{future}}{h[-1]u[n+1]} + \underbrace{h[0]u[n] + h[1]u[n-1] + h[2]u[n-2] + \dots}_{\text{present + past}}$$

For a causal system, we can't have this

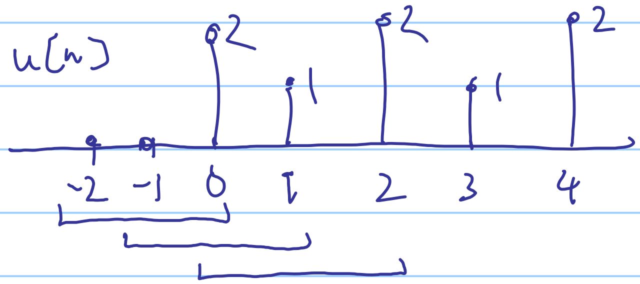
~~or in~~ In other words, a causal system must have

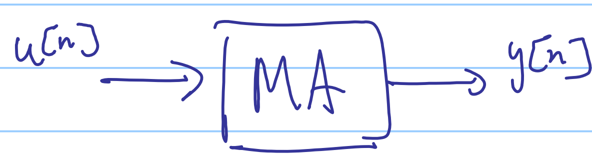
$$h[n] \equiv 0 \qquad n<0$$



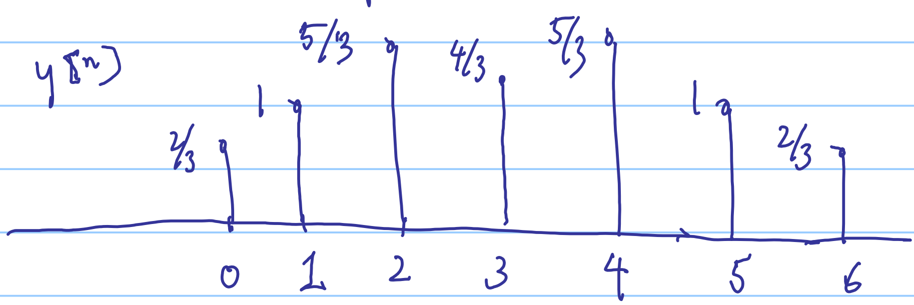
Must be identically zero, otherwise the system can anticipate the arrival of the delta function at time 0.

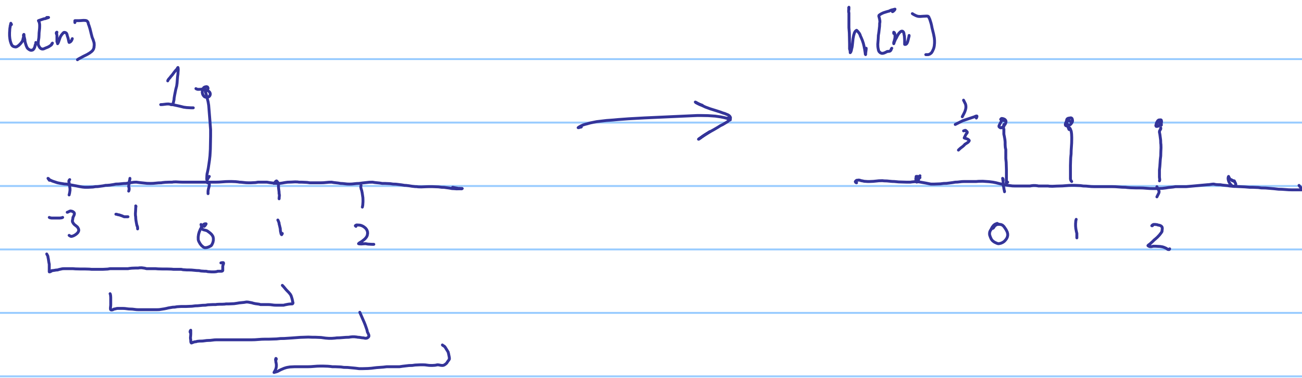
Example Moving average



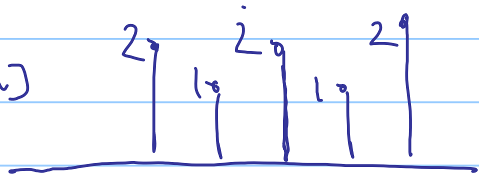


Use 3-window input





$u[n]$



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

① Fix an "n" $\rightarrow$ $y[n]$

② Draw $h[k]$ and $u[n-k]$ on the time axis in k

③ Compute $\displaystyle\sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$