Today — graphical computation of convolution
- Finite Difference Equation (DE)
- FIR, IIR
- CT convolution

To summarize the graphical computation of convolution:

1. Write out \( y[n] = \sum_{k=-\infty}^{\infty} w[n-k] \cdot h[k] \)

2. Fix an \( n \)

3. Draw \( h[k] \) and \( w[n-k] \) on the k-axis

4. Multiply \( h[k] \cdot w[n-k] \) for each \( k \)

5. Compute \( y[n] = \sum_{k} h[k] \cdot w[n-k] \)

6. Move on to the next \( n = n+1 \)

7. Go back to 2

\[
y[n] = \sum_{k=-\infty}^{\infty} w[n-k] \cdot h[k]
\]

Moving average \( h[k] = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}] \uparrow_{k=0} \)

\[ = \frac{1}{3} u[n] + \frac{1}{3} u[n-1] + \frac{1}{3} u[n-2] \]

2 registers

\[ y[n=0] = \frac{1}{3} u[0] + \frac{1}{3} u[-1] + \frac{1}{3} u[-2] \]

2 registers

\[ y[n=1] = \frac{1}{3} u[1] + \frac{1}{3} u[0] + \frac{1}{3} u[-1] \]

\[ y[2] = \frac{1}{3} u[2] + \frac{1}{3} u[1] + \frac{1}{3} u[0] \]

**Difference equation** — a single equation that relates the input and output

\[ y[n] = a_0 u[n] + a_1 u[n-1] + a_2 u[n-2] + \ldots + a_M u[n-M] \]

M registers

Order of DE = \# of registers you need to implement the system

If you have a finite duration impulse response (FIR) then convolution = DE

How about the case when h[k] is infinite "Infinite duration Impulse Response (IIR)"?
Ex/ \[ h[k] : \quad \{1 \ 1 \ 1 \ 1 \ \ldots \} \quad \rightarrow k \]

We can use graphical method if the input \( u[n] \) is finite duration.

- Overlapping is always finite # of terms.

But what if your input is also infinite?

\[ \Rightarrow \# \ of \ overlapping \ terms \ is \ growing \]

\[ \Rightarrow \# \ of \ registers \ you \ need \ will \ be \ growing \ as \ well \]

\[ y[n] = \sum_{k=-\infty}^{\infty} u[n-k] \cdot h[k] \]

\[ h[k] = \{1 \ 1 \ 1 \ \ldots \} \quad \uparrow k = 0 \]

\[ = \sum_{k=0}^{\infty} u[n-k] \]

\[ = u[n] + \sum_{k=1}^{\infty} u[n-k] \quad \Rightarrow k = 0 \]

\[ \Rightarrow k = 0 \]

\[ n-k = n-(k+1) \]

\[ = u[n] + \sum_{k=0}^{\infty} u[(n-1)-k] \]

\[ = u[n] + y[n-1] \quad \Rightarrow \text{"accumulator"} \]

Is certainly Implementable

\[ \text{Difference Equation (Recursive)} \]
Recursive

\[ y[n] - y[n-1] = u[n] \]

(output)

VS. Convolution which needs infinitely many registers \( \lim \)

Order of this DE = \# of registers = 1

General DE example:


\# of registers = 3

Is this the min \# of registers?

No: \( \min \# \text{ of register} = 2 = \max (\text{recursive non-recursive}) \)

No. \( \min \# \text{ of register} = 2 = \max (\text{recursive non-recursive}) \)

Introduce \( v[n] \)

\[ \begin{cases} v[n] - 3v[n-1] - 4v[n-2] = u[n] \\ y[n] = v[n] + v[n-1] \end{cases} \]

exactly

the same output

Register

\( \text{Share the same register.} \)