Lecture 13

Today
- graphical computation DT convolution
- Finite Diffrence Equation (DE)
- FIR, IIR
- CT convolution

To summarize the graphical computation of convolution

① Write out $y[n] = \sum_{k=-\infty}^{\infty} u[n-k] \cdot h[k]$

② Fix an n

③ Draw h[k] and u[n-k] on the k-axis

④ Multiply $h[k] \cdot u[n-k]$ for each k

⑤ compute $y[n] = \sum_k h[k]\, u[n-k]$

⑥ Move on to the next $n = n+1$

⑦ Go back to ③

$$y[n] = \sum_{k=-\infty}^{\infty} u[n-k]\, h[k]$$

Moving average $h[k] = [\underset{\overset{\uparrow}{k=0}}{\tfrac{1}{3}} \quad \tfrac{1}{3} \quad \tfrac{1}{3}]$

$$y[n] = u[n-0]\,h[0] + u[n-1]\,h[1] + u[n-2]\,h[2]$$

$$= \frac{1}{3}u[n] + \frac{1}{3}u[n-1] + \frac{1}{3}u[n-2]$$

2 Registers

$$y[n=0] = \frac{1}{3}u[0] + \frac{1}{3}u[-1] + \frac{1}{3}u[-2]$$

2 registers

$$y[n=1] = \frac{1}{3}u[1] + \frac{1}{3}u[0] + \frac{1}{3}u[-1]$$

$$y(2) = \frac{1}{3}u[2] + \frac{1}{3}u[1] + \frac{1}{3}u[0]$$

Difference equation — a single equation that relats the input and output

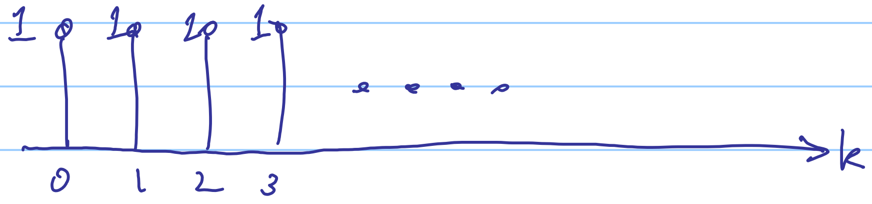
$$y[n] = \underset{\underset{\text{current input}}{\uparrow}}{a_0 u[n]} + \underbrace{a_1 u[n-1] + a_2 u[n-2] + \dots + a_M u[n-M]}_{M \text{ registers}}$$

Order of DE = minimum # of registers you need to implement the system

If you have a finite-duration impulse respone (FIR) then convolution = DE

How abut the case when $h[k]$ is infinite "Infinite duration Impulse Response (IIR)" ?

Ex/ $h[k]$:



We can use graphical method if the input $u[n]$ is finite duration

∵ overlapping is alway finite # of terms

But what if your input is also infinite?

⇒ # of overlapping terms is growing

⇒ # of registers you need will be growing as well

$h[k] = [1 \quad 1 \quad 1 \quad \cdots \quad]$
$\uparrow_{k=0}$

$$y[n] = \sum_{k=-\infty}^{\infty} u[n-k]\, h[k]$$

$$= \sum_{k=0}^{\infty} u[n-k]$$

$$= u[n] + \sum_{k=1}^{\infty} u[n-k]$$

$k' = k-1$

$k=1 \Rightarrow k'=0$

$k=\infty \Rightarrow k'=\infty$

$n-k \Rightarrow n-(k'+1)$
$=(n-1)-k'$

$$= u[n] + \sum_{k'=0}^{\infty} u[(n-1)-k']$$

$$= u[n] + y[n-1]$$

"accumulator"

Is certainly Implementable

Difference Equation (Recursive)

Need only one register !!

$$\underbrace{y[n] - \overset{\text{recursive}}{\downarrow \atop y[n-1]}}_{\text{output}} = u[n]$$

vs. Convolution which needs infinitely many registers !!!

Order of this DE $=$ min # of registers $= 1$

General DE example :

$$y[n] - 3y[n-1] - 4y[n-2] = u[n] + u[n-1]$$

# of registers = 3

Is this the min # of registers ?

No : min # of register = 2 = ~~min~~ max $\Big(\overset{2}{\Downarrow \atop \text{recursive register}}, \overset{1}{\Downarrow \atop \text{non-recursive registers}}\Big)$

share the same register.

Introduce $v[n]$

$$\text{exactly the same output} \Leftarrow \begin{cases} v[n] - 3\underset{\text{Register}}{v[n-1]} - 4\underset{\text{Register}}{v[n-2]} = u[n] \\ y[n] = v[n] + \underset{\text{Register}}{v[n-1]} \end{cases}$$