Lecture 14

Note Title 9/24/2007

Topics
- order of difference equation
- convolution vs. difference equation
- example on computing impulse response from DE
- CT convolution
- CT – Differential Equation

Examples of Recursive DE

Accumulator : $y[n] = y[n-1] + u[n]$

How do we generate impulse response $h[n]$ given DE ?

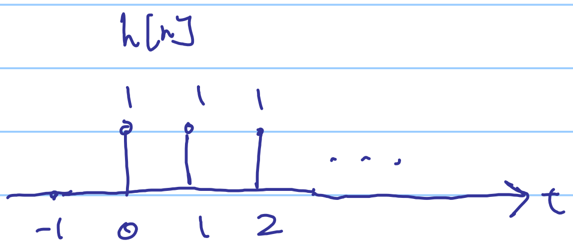
Set ① $u[n] = \delta[n]$

② Initial state is zero —— $h[n]$ is the zero-state output

$\Rightarrow \quad y[-1] = 0$

input   output

n	$u[n]$	$y[n] = y[n-1] + u[n]$
-1	0	0
0	1	$+ = 1$
1	0	$+ = 1$
2	0	$+ = 1$
3	0	1
$\vdots$		$\vdots$



Ex2 $$y[n] = \underbrace{u[n] + u[n-1]}_{\text{Input}} + \underbrace{3y[n-1] + 4y[n-2]}_{\text{Two prior output}}$$

$$\underset{\substack{\uparrow \\ \text{Register}}}{y[n] - 3y[n-1]} - \underset{\substack{\uparrow \\ \text{Register}}}{4y[n-2]} = \underset{\substack{\uparrow \\ \text{Register}}}{u[n] + u[n-1]}$$ "Direct" Form I

Is the order = 3 ? No

Order = max [memory registers required on the output $\to 2$,
memory registers required on the input $\leftarrow 1$]

= 2

The trick is to introduce an intermediate signal $v[n]$

$$\begin{cases} v[n] - 3v[n-1] - 4v[n-2] = u[n] \\ y[n] = v[n] + v[n-1] \end{cases}$$ "Direct" Form II

$\Rightarrow$ Need 2 registers only : $v[n-1]$ and $v[n-2]$

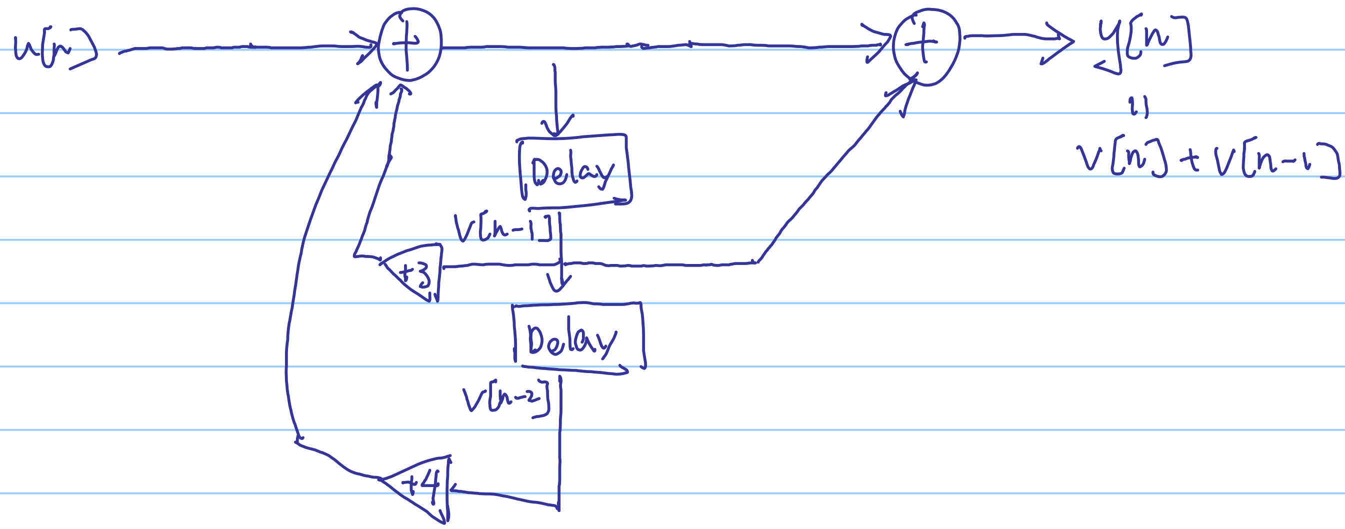
Pf:

$$\begin{aligned} y[n] &= v[n] + v[n-1] \\ -3y[n-1] &= -3v[n-1] - 3v[n-2] \\ +)\ -4y[n-2] &= -4v[n-2] - 4v[n-3] \end{aligned}$$

$$\begin{aligned} y[n] - 3y[n-1] - 4y[n-2] &= v[n] - 2v[n-1] - 7v[n-2] - 4v[n-3] \\ &= (v[n] - 3v[n-1] - 4v[n-2]) \to u[n] \\ &\quad + (v[n-1] - 3v[n-2] - 4v[n-3]) \to u[n-1] \end{aligned}$$

$$\Rightarrow \; y[n] - 3y[n-1] - 4y[n-2] = u[n] + u[n-1]$$

$$v[n] = u[n] + 3v[n-1] + 4v[n-2]$$



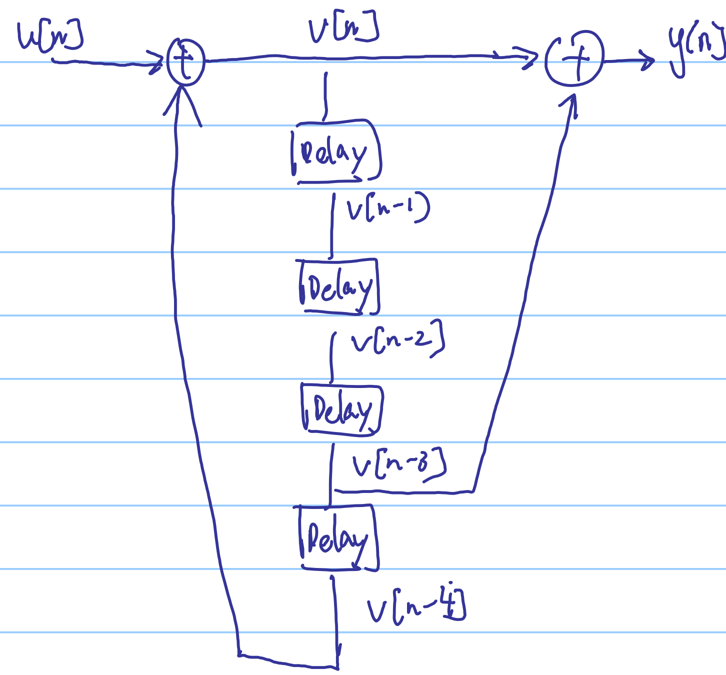
ex/

$$y[n] - y[n-4] = u[n] + u[n-3]$$

~~Two~~	
One	0
Two	5
Three	1
Four	2
>4	0

Convince Yourself

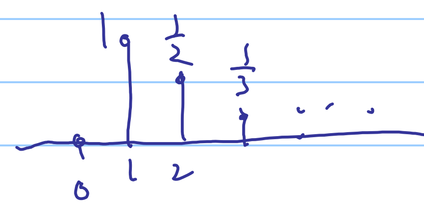
$$\begin{cases} v[n] - v[n-4] = u[n] \\ y[n] = v[n] + v[n-3] \end{cases}$$



Two ways to compute the output of a DT LTI system:

	Convolution	DE
	$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$	$y[n] - y[n-1] = u[n] + u[n-3]$
① Implementation	BAD for IIR	GOOD
② Non-zero Initial State	Cannot handle	$y[-1] = 314$ ex
③ Generality	Works for all LTI systems	Only works for Lumped LTI system

Ex/ $h[k] = \frac{1}{k}$ $\quad k > 0$



$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} u[k] \frac{1}{n-k}$$

Assume my input $u[k]$ starts at $k=0$

$$= \sum_{k=0}^{n-1} u[k] \frac{1}{n-k}$$

$$y[n+1] = \sum_{k=0}^{n} u[k] \frac{1}{(n+1)-k} = u[n+1] + \underbrace{\sum_{k=0}^{n-1} u[k] \frac{1}{(n+1)-k}}$$

— Not a lumped sys!!

cannot be expressed using $y[n], y[n-1], y[n-2] \ldots$