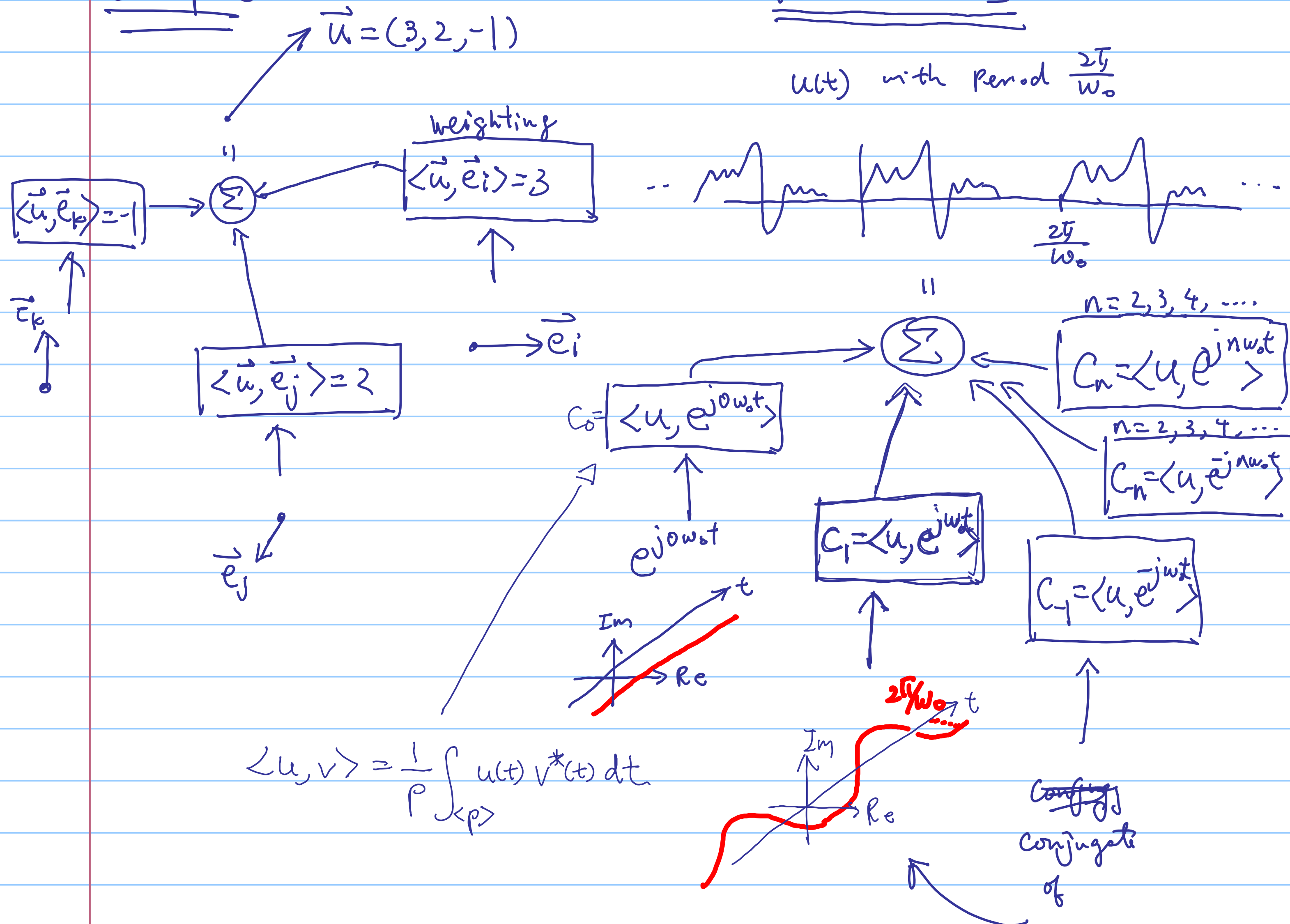
Lecture 18

Note Title 10/12/2007

- Properties Fourier Series
- Example Fourier Series
- Fourier Transform

3-D space | Fourier Series



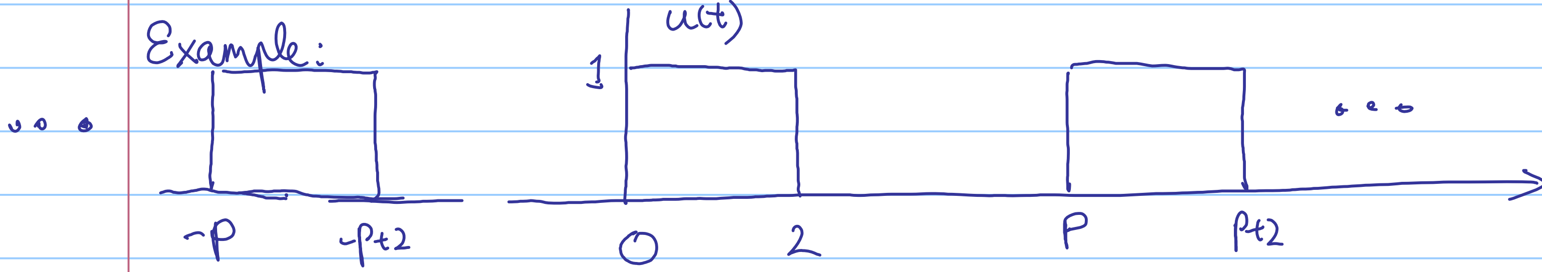
For a periodic signal $u(t)$, with fundamental freq ω_0

$$u(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

conjugate

$$C_n = \langle u, e^{jn\omega_0 t} \rangle \triangleq \frac{\omega_0}{2\pi} \int_{\langle P \rangle} u(t)\, e^{-jn\omega_0 t}\, dt$$

Example:



Want to calculate Fourier series coefficient "C_k" $k = 0, \pm 1, \pm 2, \ldots$

$$\omega_0 = 2\pi / p$$

$$C_k = \langle u(t), e^{jk\omega_0 t} \rangle$$

$$= \frac{1}{p} \int_0^p u(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{p} \int_0^2 e^{-jk\omega_0 t}\, dt$$

For $k=0$

$$\frac{1}{p} \int_0^2 \cancel{e^0}^{\,1}\, dt$$

$$= 2/p$$

$$= \frac{1}{p} \frac{1}{-jk\omega_0} \int_0^2 e^{-jk\omega_0 t}\, d(-jk\omega_0 t) \quad k \neq 0$$

$$= \frac{1}{pk\omega_0} \left(\frac{1}{-j}\right) \left[e^{-jk\omega_0 t} \right]_0^2$$

$$= \frac{1}{pk\omega_0} \left(\frac{1}{-j}\right) \left[e^{-jk\omega_0 2} - 1 \right]$$

$$= \frac{e^{-jk\omega_0}}{pk\omega_0} \left(\frac{1}{-j}\right) \left[e^{-jk\omega_0} - e^{+jk\omega_0} \right]$$

$$= \frac{e^{-jk\omega_0}}{pk\omega_0} (2 \sin k\omega_0) = \frac{2 \sin k\omega_0}{pk\omega_0} \cdot e^{-jk\omega_0}$$

Frequency Spectrum for FS : C_k vs $\omega = k\omega_0$

Amplitude (Magnitude) spectrum

phase spectrum

$|C_k|$ vs $\omega = k\omega_0$ $\qquad$ $\angle C_k$ vs $\omega = k\omega_0$

$$C_k = \begin{cases} 2/p & k=0 \\ \dfrac{2\sin(k\omega_0)}{pk\omega_0}\, e^{-jk\omega_0} & k \neq 0 \end{cases}$$

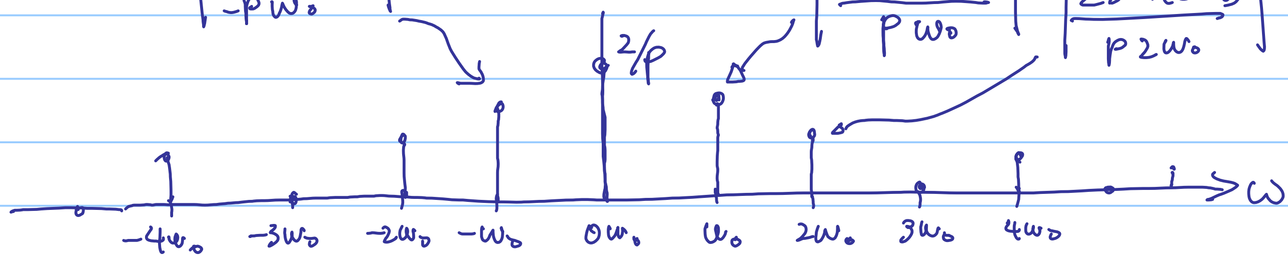
phase

$C = re^{j\theta}$

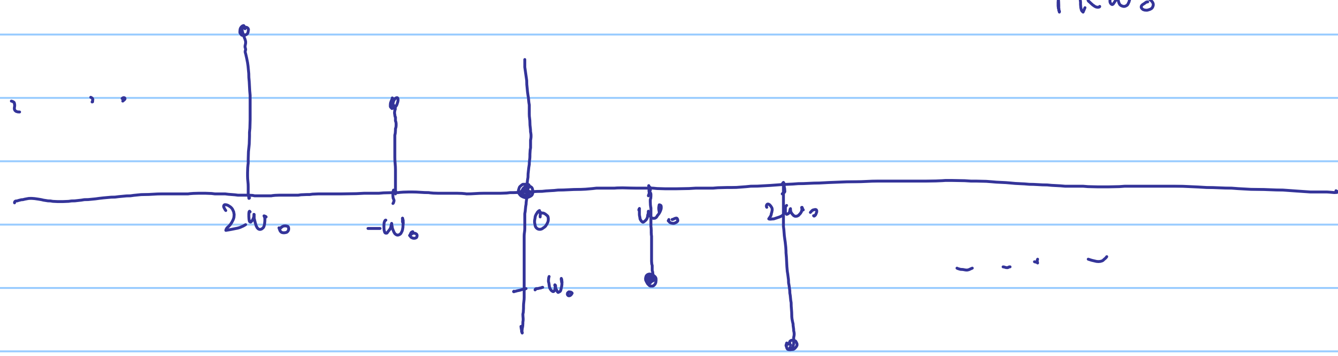
magnitude

Magnitude $\quad |C_k| = \begin{cases} 2/p & k=0 \\ \left|\dfrac{2\sin(k\omega_0)}{pk\omega_0}\right| & k \neq 0 \end{cases}$

$\left|\dfrac{2\sin(-\omega_0)}{-p\omega_0}\right| \leftarrow$ same $\rightarrow \left|\dfrac{2\sin(\omega_0)}{p\omega_0}\right|$ $\qquad \left|\dfrac{2\sin(2\omega_0)}{p2\omega_0}\right|$



Phase $\quad \angle C_k = \begin{cases} 0 & k=0 \\ -k\omega_0 & k \neq 0, \ \dfrac{2\sin(k\omega_0)}{pk\omega_0} > 0 \\ -k\omega_0 \pm \pi & k \neq 0, \ \dfrac{2\sin(k\omega_0)}{pk\omega_0} < 0 \end{cases}$



Properties

① if $u(t)$ is a real-value signal,

$$C_n = \frac{1}{P}\int_{\langle P\rangle} u(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{P}\int_{\langle P\rangle} u(t)\left[\cos(n\omega_0 t) - j\sin(n\omega_0 t)\right] dt$$

$$= \underbrace{\frac{1}{P}\int_{\langle P\rangle} u(t)\cos(n\omega_0 t)\, dt}_{\text{Real}} \underbrace{- j\,\underbrace{\frac{1}{P}\int_{\langle P\rangle} u(t)\sin(n\omega_0 t)\, dt}_{\text{real}}}_{\text{Imaginary}}$$

$$C_{-n} = \frac{1}{P}\int_{\langle P\rangle} u(t)\, e^{jn\omega_0 t}\, dt$$

$$= \frac{1}{P}\int_{\langle P\rangle} u(t)\cos(n\omega_0 t)\, dt + j\frac{1}{P}\int_{\langle P\rangle} u(t)\sin(n\omega_0 t)\, dt$$

$\Rightarrow \quad C_n = C_{-n}^{*} \quad$ or $\quad |C_n| = |C_{-n}|$

$\qquad\qquad\qquad\qquad\qquad\quad \angle C_n = -\angle C_{-n}$

② Energy of $u(t) = \langle u(t), u(t) \rangle$

$$= \frac{1}{P}\int_{\langle P\rangle} u(t)\, u^{*}(t)\, dt$$