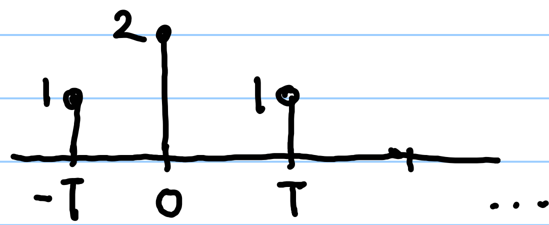
Lecture 26

Note Title 11/2/2007

- New due date for HW9 = next Wednesday
- New office hour Thurs 1–5pm

Topics
- one more example on DTFT
- model of sampling ⇒ Nyquist Sampling Thm
- Inverse DTFT

ex/ Find $X_d(\omega)$ of



$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega nT}$$

$$= \cancel{x[-1]}^{\,1} e^{-j\omega(-T)} + \cancel{x[0]}^{\,2} e^{-j\omega 0T} +$$

$$\cancel{x[1]}^{\,1} e^{-j\omega T}$$

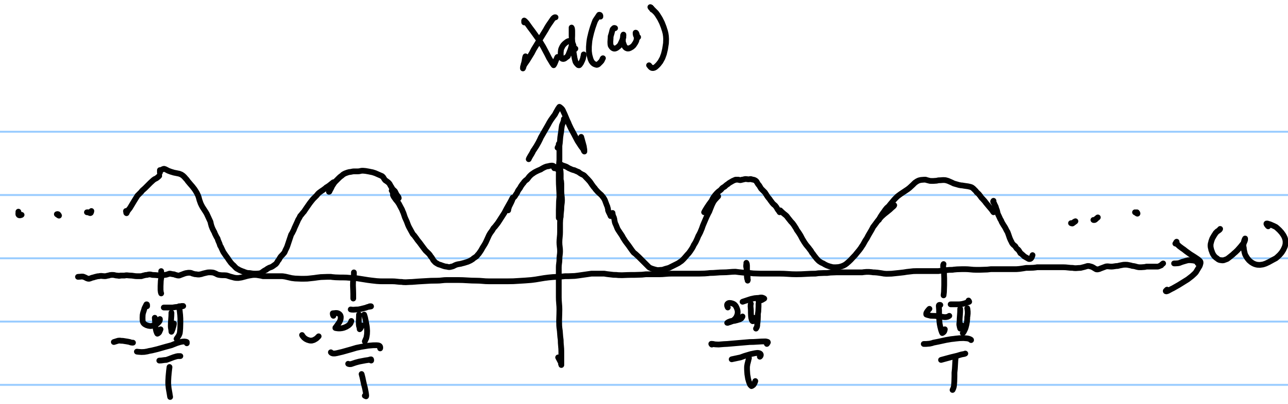
$$= e^{j\omega T} + 2 + e^{-j\omega T}$$

$$= 2\cos(\omega T) + 2$$

Frequency Spectrum

variable

constant = sampling period



Is the fact $X_d(\omega)$ being periodic a special property of $x[n]$ or is it more general??

$X_d(\omega)$ is a weighted sum of $\underline{\underline{e^{-j\omega nT}}}$

This is periodic in both n and ω !!!

higher frequency

$$e^{-j\omega nT} \qquad\qquad e^{-j(\omega + k\frac{2\pi}{T})nT}$$

when k is an integer

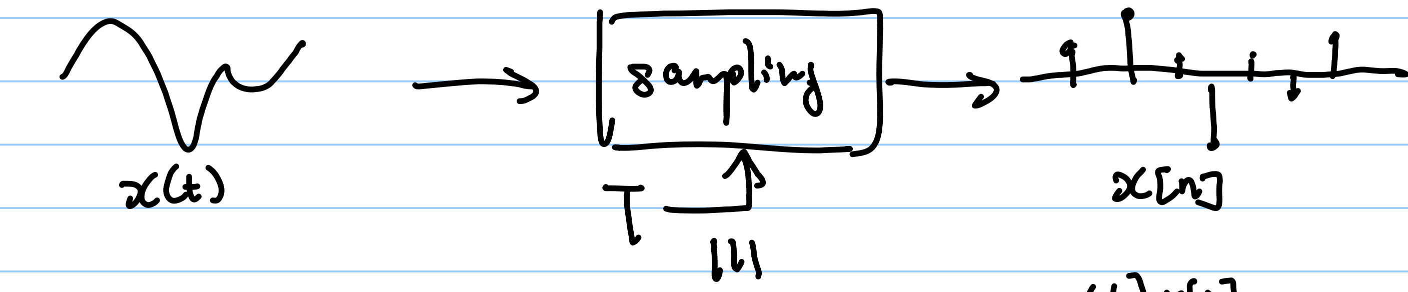
$$= e^{-j\omega nT - jk\frac{2\pi}{\cancel{T}}n\cancel{T}}$$

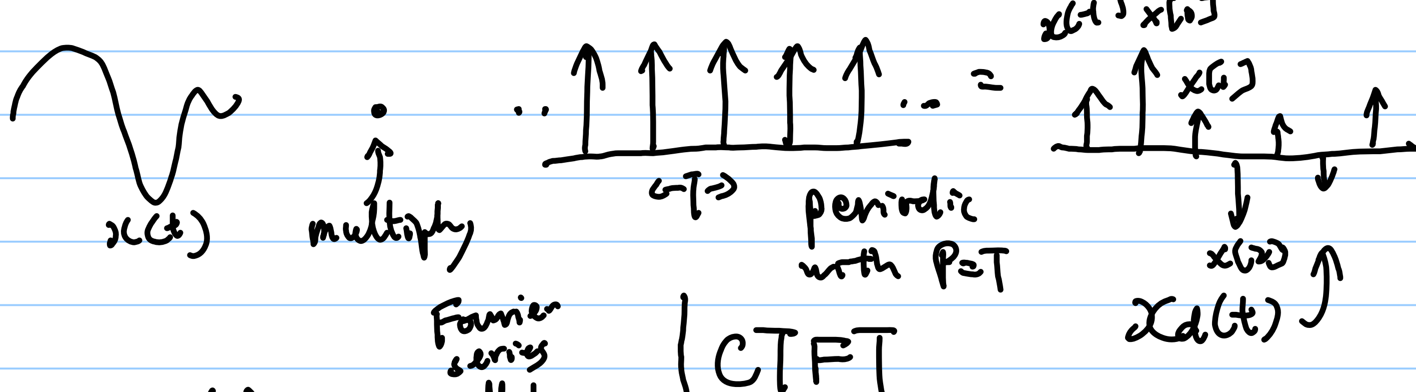
$$= e^{-j\omega nT} \cdot \cancel{e^{-jnk2\pi}} \rightarrow 1$$

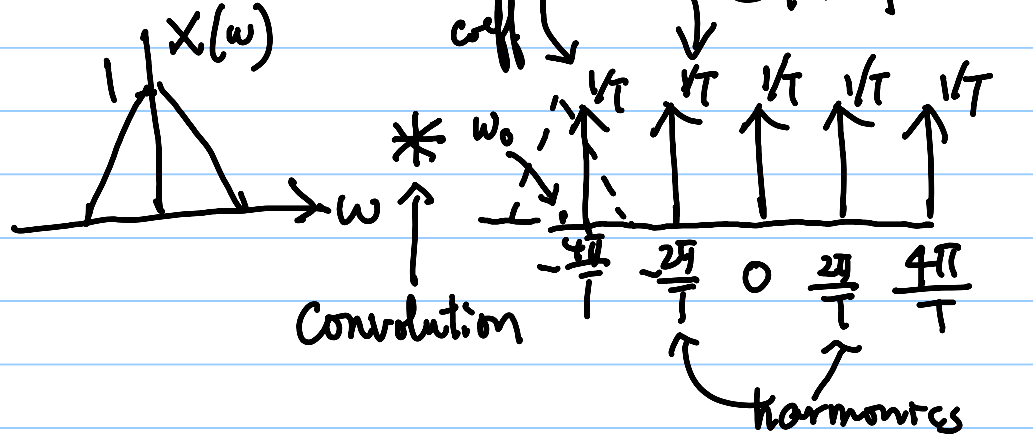
$$= e^{-j\omega nT}$$

$\Rightarrow X_d(\omega)$ must be periodic in ω with period $= \frac{2\pi}{T}$

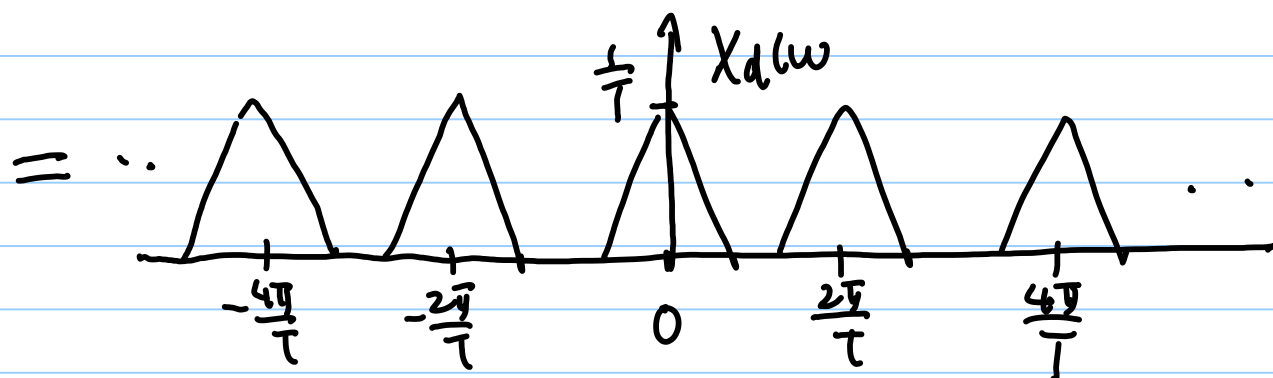
Sampling Process ⇒ Better interpretation of $X_d(\omega)$







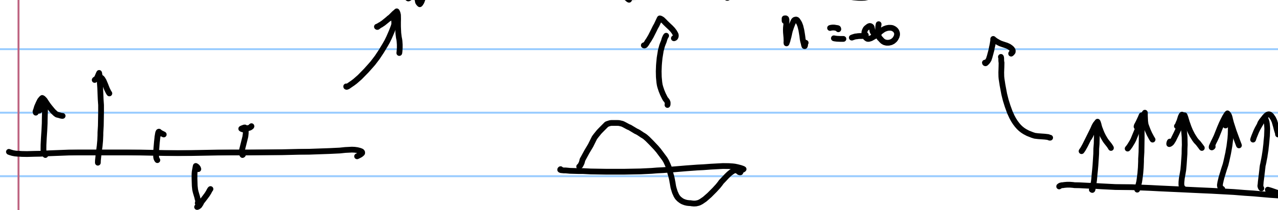
$$\int X(\omega_0-\omega)\,\frac{1}{T}\,\delta\!\left(\omega+\frac{4\pi}{T}\right)d\omega = \frac{1}{T}X\!\left(\omega_0+\frac{4\pi}{T}\right)$$



– assume  overlaps at most one delta function

$X_d(\omega)$ is the replication of $X(\omega)$ at every $\omega = \frac{2\pi}{T}$ interval.

$$x_d(t) = \underline{x(t)} \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT)$$



$$X_d(\omega) = \mathcal{F}[x_d(t)] = X(\omega) * \mathcal{F}\left[\sum_{n=-\infty}^{\infty} \delta(t-nT)\right]$$

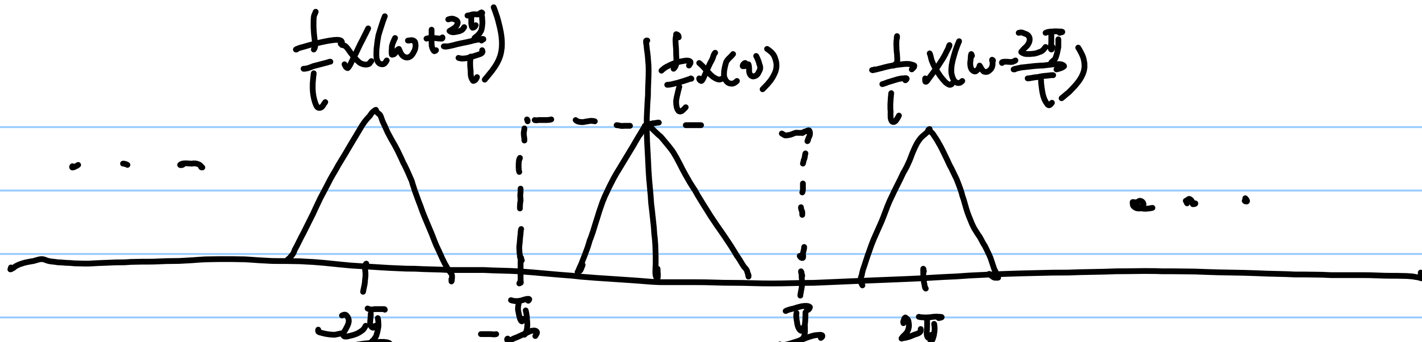
$$= X(\omega) * \sum_{k=-\infty}^{\infty} \frac{1}{T}\,\delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$= \int X(\omega-\lambda)\left[\sum_{k=-\infty}^{\infty} \frac{1}{T}\,\delta\left(\lambda - k\frac{2\pi}{T}\right)\right] d\lambda$$

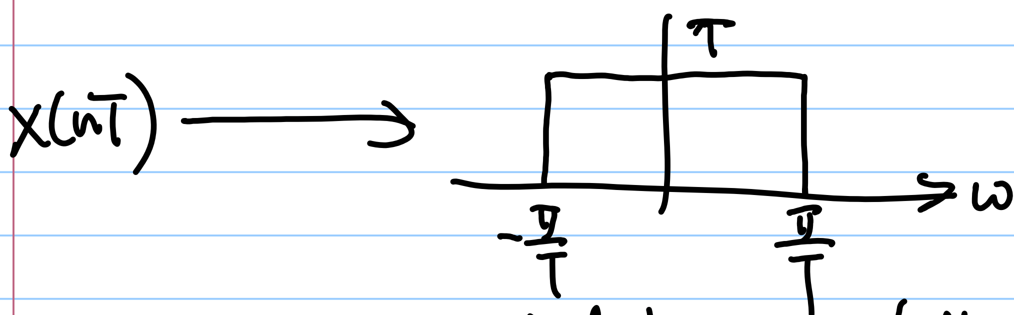
$$= \sum_{k=-\infty}^{\infty} \frac{1}{T} \int X(\omega-\lambda)\,\delta\left(\lambda - k\frac{2\pi}{T}\right) d\lambda$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{T}\, X\left(\omega - k\frac{2\pi}{T}\right)$$

$$= \ldots + \underbrace{\frac{1}{T} X\left(\omega + \frac{2\pi}{T}\right)}_{(k=-1)} + \underbrace{\frac{1}{T} X(\omega)}_{(k=0)} + \underbrace{\frac{1}{T} X\left(\omega - \frac{2\pi}{T}\right)}_{(k=1)} + \ldots$$

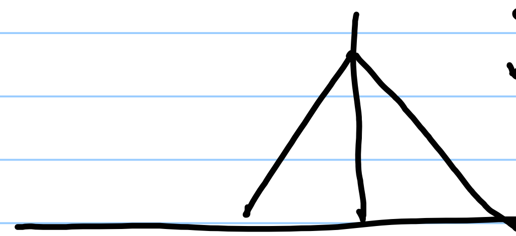


— each period resembles the original $X(\omega)$



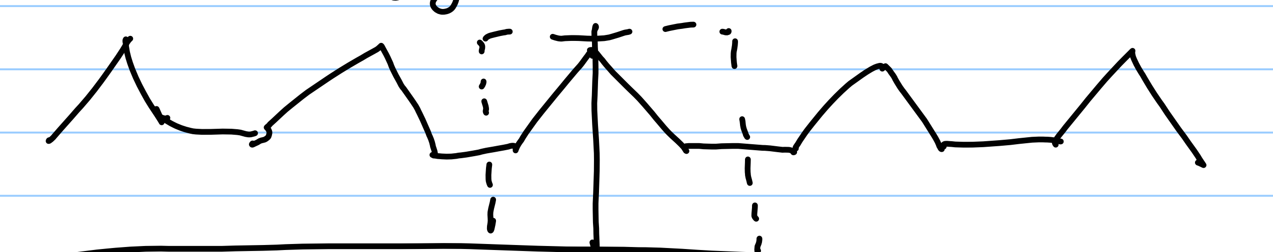
Ideal Low pass filter

$\hookrightarrow$



Spectrum of the original signal !!!

What if your sampling rate is not high enough (T is too big)?



$\Rightarrow$ sampling has to be high enough

$B > \frac{2\pi}{T}$ Next time