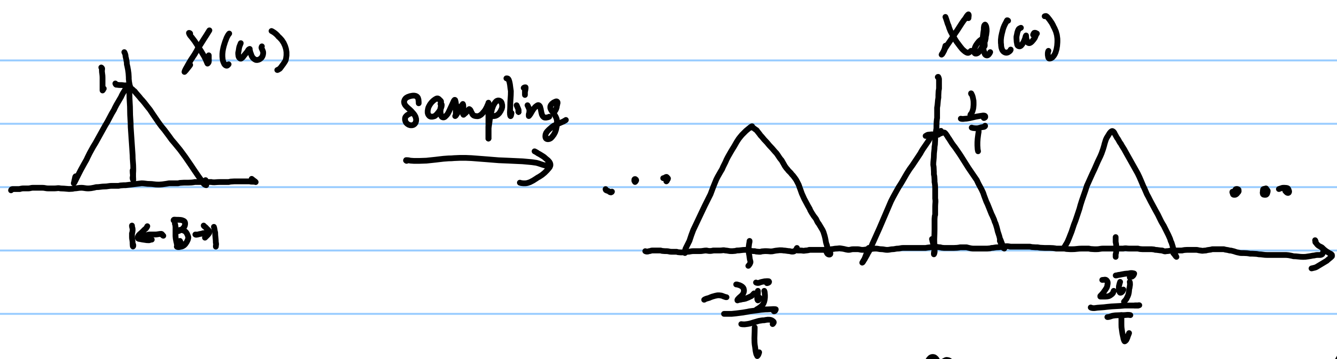
Lecture 27

Note Title 11/5/2007

Today :
- Nyquist Sampling Theorem
- Practical D/A methods
- Inverse DTFT
- Discrete Fourier Transform
- Overview on Laplace , Z transform



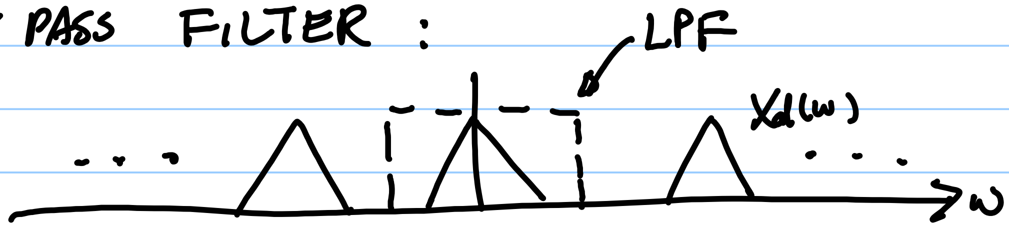
$$X_d(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\omega - k\frac{2\pi}{T}\right)$$

Bandwidth = highest frequency in $X(\omega)$ beyond which all the frequency components of $X(\omega)$ become zero.

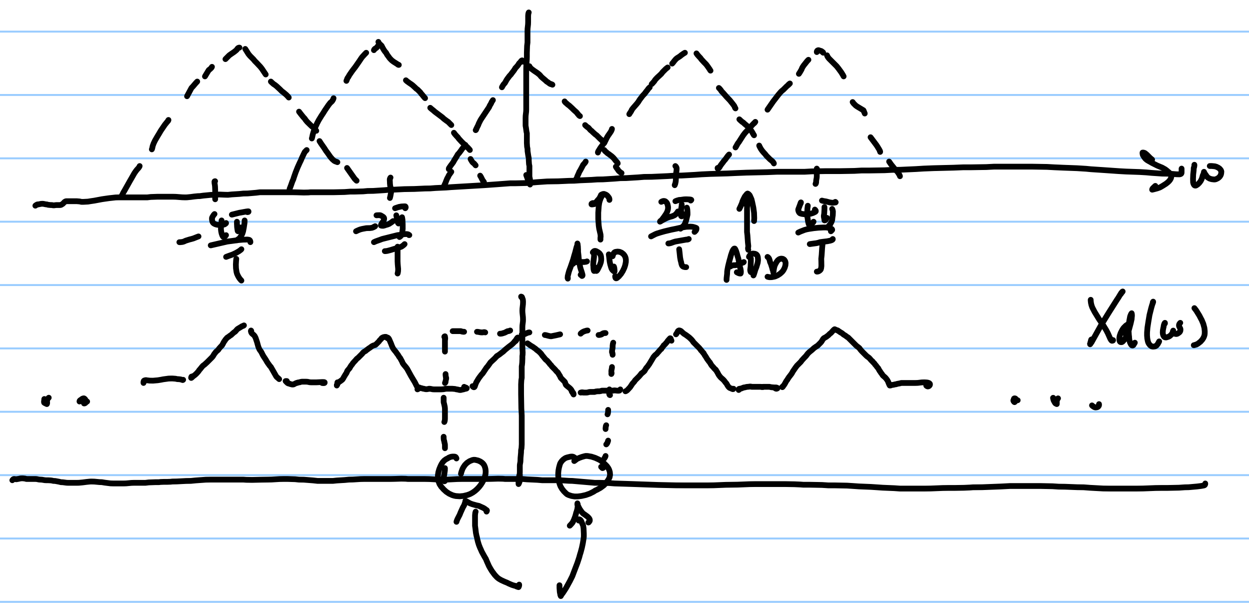
Case 1 / $B \leq \frac{\pi}{T}$ (In Hz, $B \leq \frac{1}{2} f_s \Leftrightarrow f_s \geq 2B$)

$\Rightarrow$ Adjacent copies in $X_d(\omega)$ do not overlap

$\Rightarrow$ the original CT signal $x(t)$ can be PERFECTLY RECONSTRUCTED from $\hat{x}(nT)$ using an IDEAL LOW PASS FILTER :



Case 2 : $B > \frac{\pi}{T}$ (in Hz $f_s < 2B$)



Corruption of mid-low frequencies by high frequency components ⇒ ALIASING

> **Nyquist Sampling Thm :**
> if $f_s \geq 2B$ ⇒ signal can be perfectly reconstructed with an ideal LPF

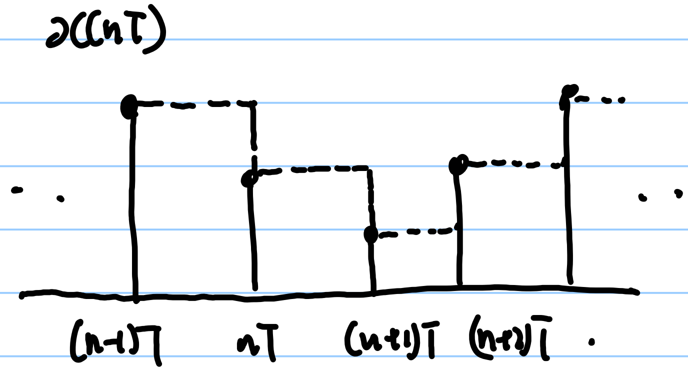
ex/ Human voice < 4000 Hz

⇒ What should be our sampling rate ? 8 kHz

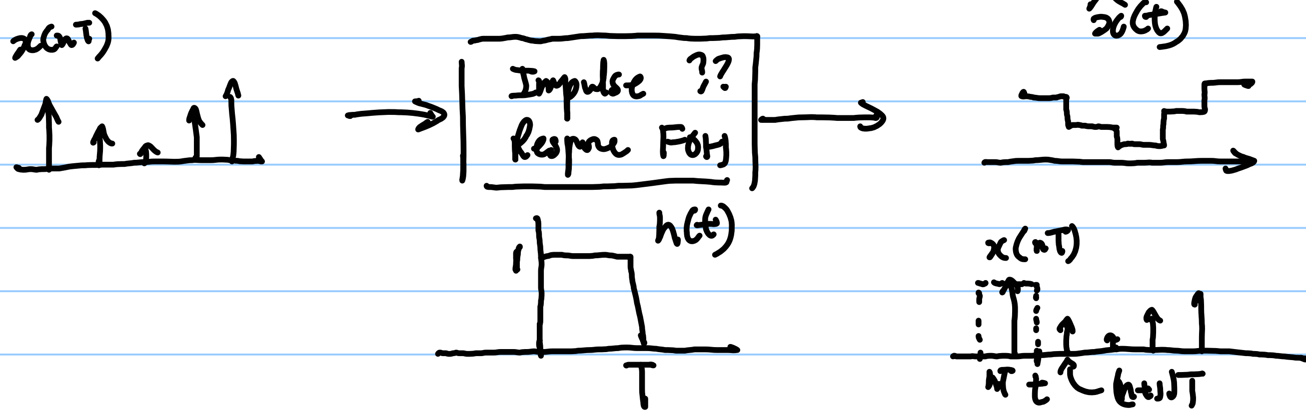
⇒ 8 bit / sample ⇒ 64 kbit/s. (Second-generation of modem)

Practical D/A conversion (getting away from not using ideal LPF)

①

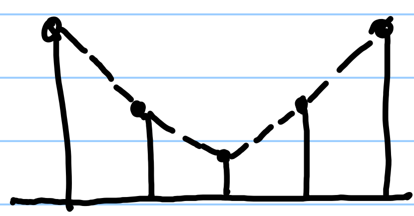


First order hold (FOH)
or
Sample-and-hold

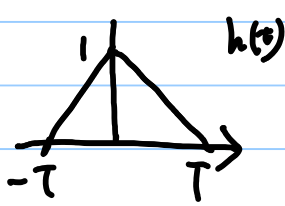


$\Rightarrow x(t) = x(nT)$

2)



Linear Interpolation



(In HW)

Inverse Discrete-Time Fourier Transform

- it's not commonly used (we'll learn a better way with z-transform)

Forward Transform : $X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega nT}$

Inverse Transform : $x[n] = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} X_d(\omega) e^{j\omega nT} d\omega$

$X_d(\omega)$ is a periodic function in ω

$P = \frac{2\pi}{T}$

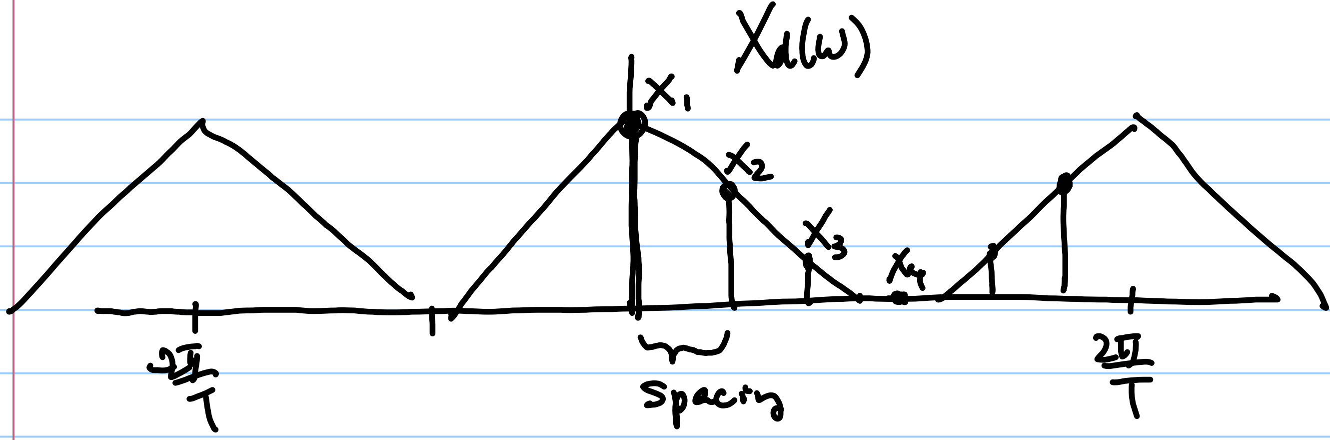
$\Rightarrow$ has a FOURIER SERIES REPRESENTATION in ω

$\Rightarrow$ $x[n]$ is the corresponding frequency coefficient

But $X_d(\omega)$ is continuous in ω

$\Rightarrow$ cannot be handled by a computer

Answer : Frequency sampling



Let's say we use N samples for one period

$$\text{Spacing} = \frac{2\pi}{TN}$$

$$X_k = X_d\left(\frac{2\pi}{TN}k\right) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jnT\cdot\frac{2\pi}{TN}k}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\frac{2\pi}{N}nk}$$

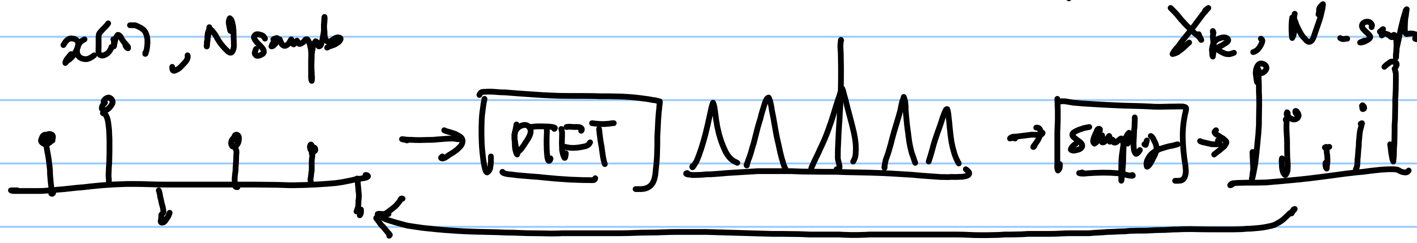
frequency index

# of frequency samples per $\frac{2\pi}{T}$ period.

Time index

Under what condition that we can perfectly reconstruct $x[n]$ using $X_0, X_1, X_2, \ldots, X_{N-1}$ ← frequency samples of $X_d(\omega)$?

If $x[n]$ is finite and has N samples !!



$\Rightarrow$ Discrete Fourier Transform (DFT)