Today:
- Nyquist Sampling Theorem
- Practical D/A methods
- Inverse DTFT
- Discrete Fourier Transform
- Overview on Laplace, Z transform

\[ X_d(w) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(w - k \frac{2\pi}{T}) \]

Bandwidth = highest frequency in \( X(w) \) beyond which all the frequency components of \( X(w) \) become zero.

Case 1 / \( B \leq \frac{\pi}{T} \) (In Hz, \( B \leq \frac{1}{2} f_s \) \( \Rightarrow f_s \geq 2B \))

⇒ Adjacent copies in \( X_d(w) \) do not overlap

⇒ the original CT signal can be PERFECTLY RECONSTRUCTED from \( x(nT) \) using an IDEAL LOW PASS FILTER : 

\[ \ldots \quad \text{LPF} \quad \ldots \]
Case 2: $B > \frac{\pi}{4}$ (in Hz $f_8 < 2B$)

Corruption of mid-low frequencies by high frequency components $\Rightarrow$ ALIASING

Nyquist Sampling Theorem:
if $f_8 > 2B$ $\Rightarrow$ signal can be perfectly reconstructed with an ideal LPF.

ex/ Human voice < 4000 Hz

$\Rightarrow$ What should be our sample rate? 8 kHz
$\Rightarrow$ 8 bit/sample $\Rightarrow$ 64 kbit/s (second-gener.)
Practical D/A conversion (giving away from not using ideal LPF)

1) Sample and hold (FOH) $x(nT)$

2) Linear Interpolation $x(nT)$

First order hold (FOH)

$x(nT)$

$\Rightarrow x(t)$ (In HW)
Inverse Discrete-Time Fourier Transform

- It's not commonly used (we'll be using a better way with Z-transform)

Forward Transform:
\[ X_d(w) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwnT} \]

Inverse Transform:
\[ x[n] = \frac{T}{2\pi} \int_{-\frac{T}{2}}^{\frac{T}{2}} X_d(w) e^{jwnT} \, dw \]

\( X_d(w) \) is a periodic function in \( w \)
\[ p = \frac{\frac{2\pi}{T}}{1} \]

⇒ has a Fourier Series Representation in \( w \)
⇒ \( x[n] \) is the corresponding frequency coefficient

But \( X_d(w) \) is continuous in \( w \)
⇒ cannot be handled by a computer

Answer: Frequency sampling
Let's say we use $N$ samples for one period.

$$\text{Spacing} = \frac{2\pi}{TN}$$

$$X_k = X_d\left(\frac{2\pi}{TN} k\right) = \sum_{n=-\infty}^{\infty} x[n] e^{-j \frac{2\pi}{TN} k}$$

Under what condition that we can perfectly reconstruct $X_d\left(\frac{2\pi}{TN} k\right)$ using $X_0, X_1, X_2, \ldots, X_{N-1}$ frequency samples of $X_d(w)$?

If $x[n]$ is finite and has $N$ samples!}

$$x[n], N \text{ samples} \rightarrow \left[\text{Overlap Add} \right] \rightarrow \sum_{k} X_k, N \text{ samples}$$
⇒ Discrete Fourier Transform (DFT)