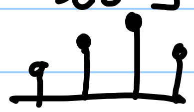
Lecture 36

Note Title 12/5/2007

z - transform

DT $x[n]$ $\longrightarrow$ z-transform

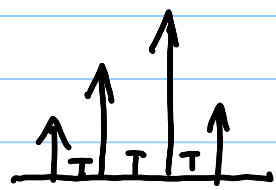


$$Z\{x(n)\} = \sum_{n=0}^{\infty} x[n] z^{-n}$$

causal sequence only

CT surrogate

$x_d(t)$

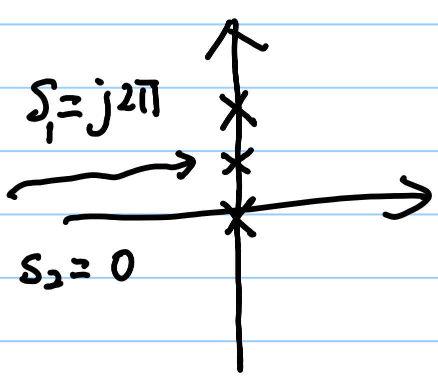


$$= \sum_{n=0}^{\infty} x[n]\,\delta(t-nT)$$

$\longrightarrow$ $\mathcal{L}\{x_d(t)\}$

$z = e^{sT}$ change of variable

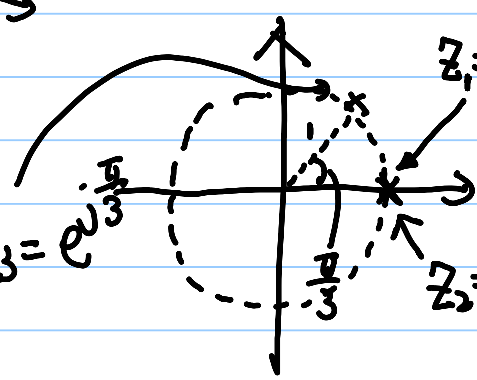
S - plane $\xrightarrow{z = e^{sT}}$ z - plane | ex $T=1$



$s_1 = j2\pi$

$s_3 = j\frac{\pi}{3}$

$s_2 = 0$



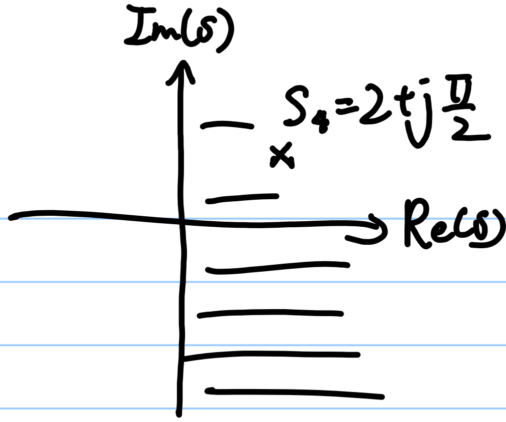
$z_1 = e^{j2\pi}$

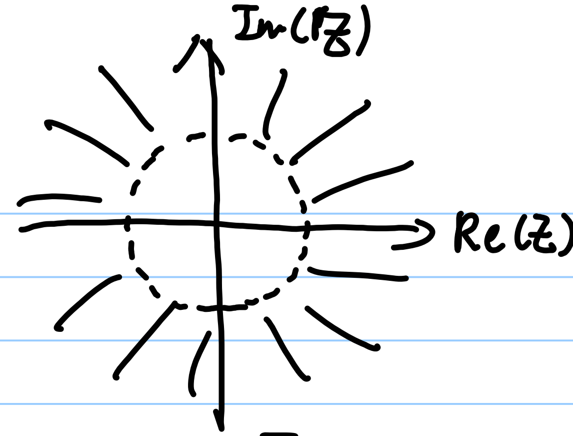
$z_3 = e^{j\frac{\pi}{3}}$

$\frac{\pi}{3}$

$z_2 = e^{j0}$

1/ $s = j\omega$ $\xrightarrow{\text{maps to}}$ unit circle in z.





$$z_4 = e^{sT}$$

Im(s_4)

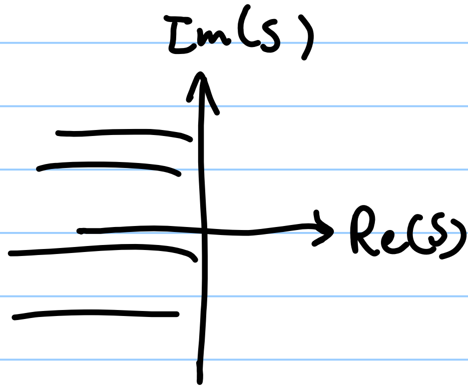
Re(s_4)

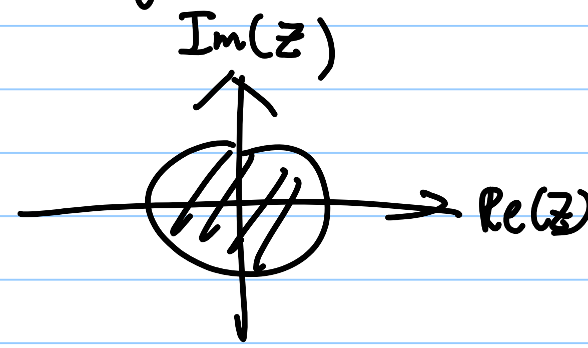
$$= e^{(2+j\frac{\pi}{2})}$$

$$= e^{2} \cdot e^{j\frac{\pi}{2}}$$

magnitude

phase



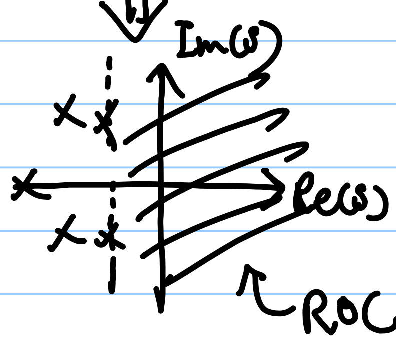


2/ $\mathcal{L}\{h_d(t)\}$ is BIBO stable

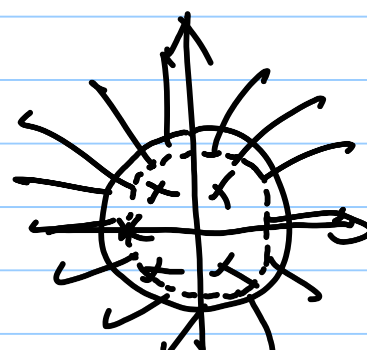
$H(z) = \mathcal{Z}\{h[n]\}$

ct surrogate of a DT impulse response of a LTI system

$\Downarrow$



$\Rightarrow$



ROC: outermost pole

$z = p$

all z with $|z| > |p|$

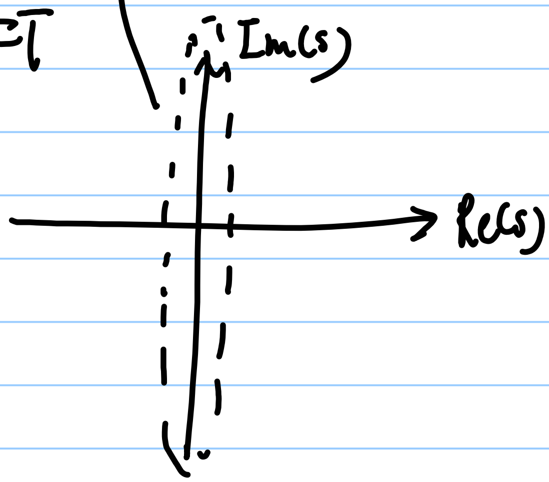
3) BIBO stable $\Leftrightarrow$ all poles are in the open LHP $\longrightarrow$ BIBO stability $\Leftrightarrow$ all poles are inside the unit circle.

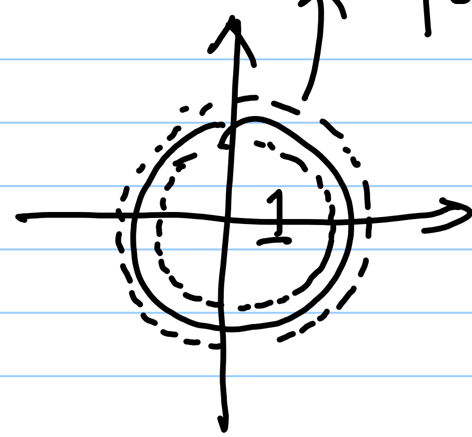
4) $X(s) = \mathcal{L}\{x(t)\}$ and ROC contains $j\omega$-axis $\longrightarrow$ $X(z) = \mathcal{Z}\{x[n]\}$ and ROC contains unit circle

$$X(\omega) = X(s)\Big|_{s=j\omega}$$

CTFT

$$X_d(\omega) = X(z)\Big|_{z=e^{j\omega T}}$$





it is a causal.

ex $\quad x[n] = \underline{\underline{\left(\frac{1}{2}\right)^n q[n]}}$

$$X_d(\omega) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n q[n]\, e^{-j\omega T n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega T n} = \frac{1}{1-\frac{1}{2}e^{-j\omega T}}$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

LTI system

Difference Equation

Accumulator $\quad y[n] = y[n-1] + u[n]$

Applying Z-transfrm:

$$y[n] \longrightarrow Y(z)$$

$$u[n] \longrightarrow U(z)$$

$$y[n-1] \longrightarrow Z\{y[n-1]\} = \sum_{n=0}^{\infty} y[n-1]\, z^{-n}$$

sub $m = n-1$

$$= \sum_{m=-1}^{\infty} y[m]\, z^{-m-1}$$

$m=-1$ term:

$$= y[-1]\, \underbrace{\cancel{z^{1-1}}}_{1} + \sum_{m=0}^{\infty} y[m]\, z^{-m} \cdot z^{-1}$$

Initial condition

$$= y[-1] + z^{-1} Y(z)$$

$$y[n] = y[n-1] + u[n]$$

$$\mathcal{Z} \Big\downarrow$$

$$Y(z) = y[-1] + z^{-1} Y(z) + U(z)$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{U(z)} \qquad Y(z) = Y_{ZS}(z) + Y_{ZI}(z)$$