Lecture 37

Note Title 12/6/2007

Z-transform
- Difference Eq
- Inverse Z-transform
- Jury test

From last time, an accumulator can be described as

$$\underbrace{y[n]}_{\text{output}} = y[n-1] + \underbrace{u[n]}_{\text{input}}$$

Z-transform → $Y(z) = \underbrace{y[-1]}_{\text{initial condition}} + z^{-1}Y(z) + U(z)$

$$(1-z^{-1})Y(z) = y[-1] + U(z)$$

$$Y(z) = \underbrace{\frac{y[-1]}{1-z^{-1}}}_{\text{Zero-Input Response}} + \underbrace{\frac{U(z)}{1-z^{-1}}}_{\text{Zero-state Response with transfer function}}$$

$$H(z) = \frac{1}{1-z^{-1}}$$

A "smoother" accumulator (why?)

$$y[n] = 0.5\,y[n-1] + 0.5\,y[n-2] + u[n]$$

$$y[n] \xrightarrow{\mathcal{Z}} Y(z)$$

$$u[n] \xrightarrow{\mathcal{Z}} U(z)$$

$$0.5\,y[n-1] \xrightarrow{\mathcal{Z}} 0.5\,y[-1] + 0.5 z^{-1} Y(z)$$

$0.5\,y[n-2] \xrightarrow{\mathcal{Z}}$ Apply the delay rule twice !

$$0.5\,y[-2] + 0.5 z^{-1} \mathcal{Z}\{y[n-1]\}$$

$$= 0.5\,y[-2] + 0.5 z^{-1} y[-1] + 0.5 z^{-1} \cdot z^{-1} Y(z)$$

$$= 0.5\,y[-2] + 0.5 z^{-1} y[-1] + 0.5 z^{-2} Y(z)$$

$$Y(z) = 0.5\,y[-1] + 0.5 z^{-1} Y(z) + 0.5\,y[-2] + 0.5 z^{-1} y[-1] + 0.5 z^{-2} Y(z) + U(z)$$

$$Y(z) = \frac{0.5\,y[-1] + 0.5\,y[-2] + 0.5 z^{-1} y[-1]}{1 - 0.5 z^{-1} - 0.5 z^{-2}} + \frac{1}{1 - 0.5 z^{-1} - 0.5 z^{-2}} U(z)$$

Common Z-transform Pairs

	$x[n]$	$X(z)$
	$\delta[n]$	1
	$\delta[n-n_0]$	z^{-n_0}
	$q[n]$	$\frac{1}{1-z^{-1}}$
$b^n q[n] \Leftrightarrow$	b^n	$\frac{1}{1-bz^{-1}}$
$n b^n q[n] \Leftrightarrow$	$n b^n$	$\frac{bz^{-1}}{(1-bz^{-1})^2}$

Negative power only $\because$ causal sequence as $X(z)=\sum_{n=0}^{\infty} x[n]\, z^{-n}$

More in text but that's all we need

Inverse Z transform

$$X(z) = \frac{2+10z^{-1}}{(1+z^{-1})(-2+z^{-1})}$$

① Turn it into the power : $X(z) = \frac{2z^2+10}{(z+1)(z-2)}$

② Write $\tilde{X}(z) = \frac{X(z)}{z} = \frac{2z^2+10}{z(z+1)(z-2)}$

③ Partial Fraction of $\tilde{X}(Z)$ (Same as LT)

$$\frac{X(z)}{z} = -5\frac{1}{z} + 4\frac{1}{z+1} + 3\frac{1}{z-2}$$

$$X(z) = -5 + 4\frac{z}{z+1} + 3\frac{z}{z-2}$$

$$= -5 + 4\frac{1}{1+z^{-1}} + 3\frac{1}{1-2z^{-1}}$$

$\frac{X(z)}{z}$ allows us to "recreate" negative power of z

④ Table look up

$$-5 \xrightarrow{z^{-1}} -5\delta[n]$$

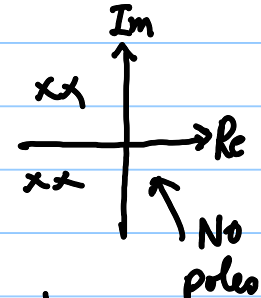
$$4\frac{1}{1+z^{-1}} \xrightarrow{z^{-1}} 4\,g[n]$$

$$3\frac{1}{1-2z^{-1}} \xrightarrow{z^{-1}} 3(-2)^n g[n]$$

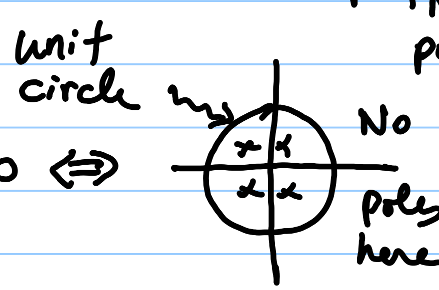
$\to \oplus \to h[n]$

Finally stability test : Jury test

In LT BIBO stable $\Leftrightarrow \int_0^\infty |h(t)|\,dt < \infty \Leftrightarrow$



In ZT BIBO stable $\Leftrightarrow \sum_{n=0}^{\infty} |h[n]| < \infty \Leftrightarrow$



How to test if $H(z) = \dfrac{N(z)}{-1+2z^{-1}-0.8z^{-3}}$ is BIBO stable?

Denominator $= -1+2z^{-1}-0.8z^{-3}$

$= z^{-3}(-z^3+2z^2-0.8)$

positive power only

$= -z^{-3}(z^3-2z^2+0.8)$

positive

LEADING POWER

Jury test
(similar to Routh)

	1	-2	0	0.8	
Reverse :	0.8	0	-2	1	$k_1 = \frac{0.8}{1} = 0.8$
	$1-k_1 0.8 = 0.36$	$-2-k_1 0 = -2$	$0-k_1(-2) = 1.6$	$0.8-k_1 1 = 0$	Drop the zero
Reverse :	1.6	-2	0.36		$k_2 = \frac{1.6}{0.36} = 4.44$
	$0.36-k_2 1.6 = -6.744$				

SUBSEQUENT LEADING COEFFICIENTS

$\Rightarrow$ any of them 0 or negative implies a root outside or on the unit circle

$\Rightarrow$ UNSTABLE