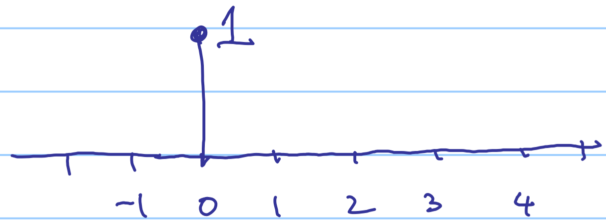
EE422: Lecture 5

Note Title 8/31/2007

Outline
1. DT Impulse — sifting
2. CT sinusoidal signals
3. DT sinusoidal signals

DT Impulse

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{everywhere else} \end{cases}$$



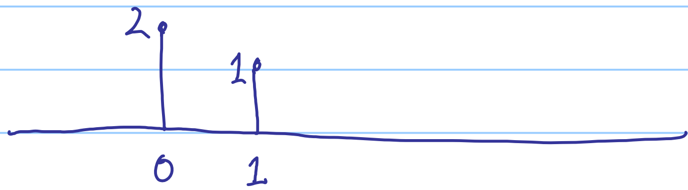
Sifting properties — every DT sequence $x[n]$ can be represented as **a weighted sum of shifted impulses**

$$x[n] = \sum_{k=-\infty}^{\infty} a_k \, \delta[n-k]$$

$$= \ldots + a_{-2}\delta[n-2] + a_{-1}\delta[n-1] + a_0\delta[n]$$
$$+ a_1\delta[n-1] + a_2\delta[n-2] + \ldots$$

It turns out $a_k = x[k]$ ~ signal value at time k

Ex/ $x[n]$



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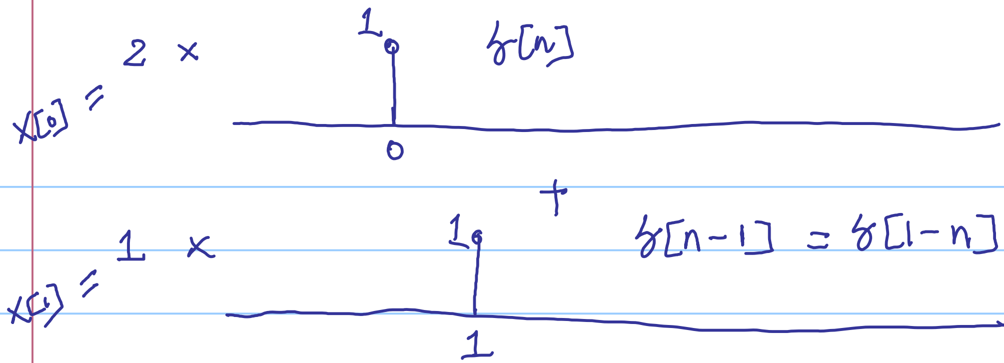


$x[n] = 2\cdot\delta[n] + 1\cdot\delta[n-1]$

$= x[0]\cdot\delta[n-0] + x[1]\cdot\delta[n-1]$

Sifting property

DT $$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

CT $$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

summation done in continuous time

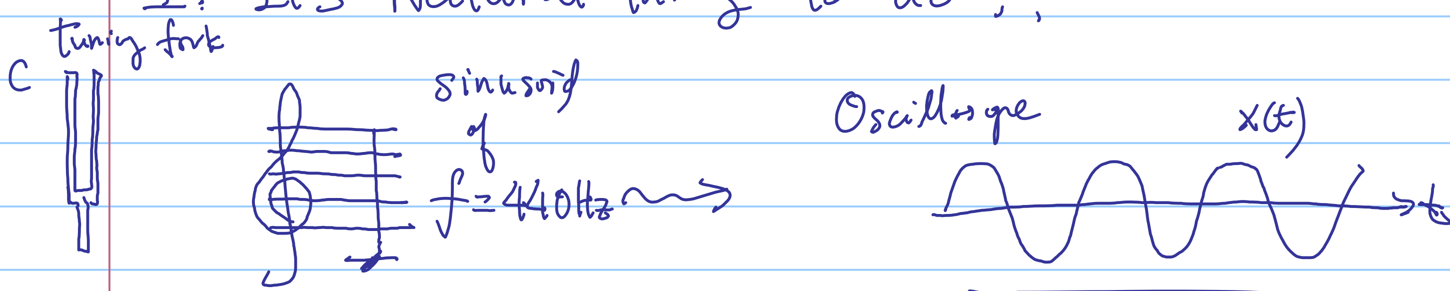
Integration in CT $\longleftrightarrow$ (Duality) Summation in DT

Sinusoidal Signals

Idea : we want to represent arbitrary signal (CT & DT) as **weighted summation (integral) of sinusoidal signals of different frequencies.**

Why sinusoidal signals?

I. It's Natural thing to do !!



Color : frequency of light   red color → 450 THz
                                           Violet — 700 THz

Texture or Edges : frequency in (x, y) "spatial"

II. Represent in Computer

Audio : MIDI, MP3
Video : JPEG, MPEG
} utilizing frequencies

↳ ① Represent a signal as a summation of many sinusoidal signals at diffrent frequencies

② Remove those sinusoidal signals whose frequencies are not important for perception.

III. Sinusoid is "appropriate" for Linear System

CT Sinusoids

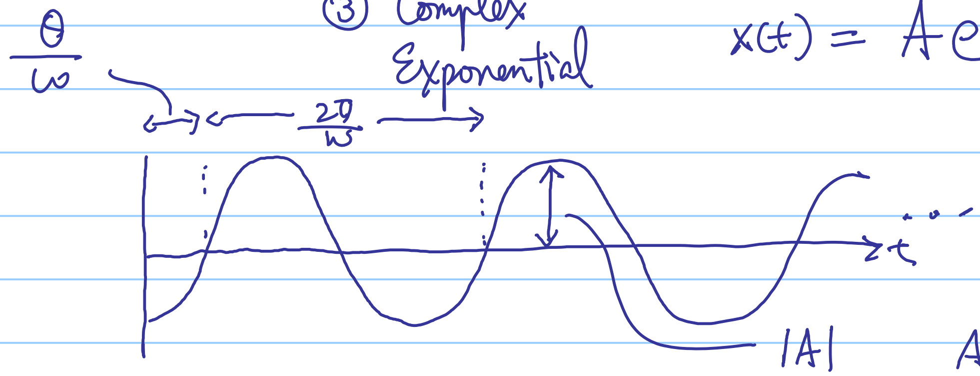
Three kinds

① Sine $x(t) = A\sin(\omega t + \theta)$

② cosine $x(t) = A\cos(\omega t + \theta)$

③ Complex Exponential $x(t) = Ae^{j(\omega t + \theta)}$

Sine



ω = angular freq (rad s^{-1})

θ = phase (rad)

A = amplitude

Cosine is similar

sine and cosine are interchangeable:

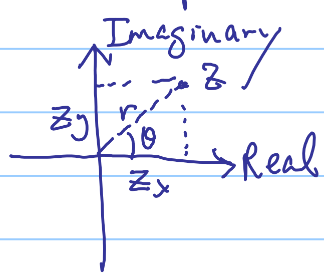
$$A\sin(\omega t+\theta) = A\cos(\omega t+\theta-\tfrac{\pi}{2})$$
$$A\cos(\omega t+\theta) = A\sin(\omega t+\theta+\tfrac{\pi}{2})$$

ω can be negative but negative sine → we can easily convert back to positive frequency.

$$\underbrace{A\sin(-\pi t)}_{\omega=-\pi} = -A\sin(\underbrace{\pi t}_{\omega=\pi \text{ +ve}})$$

$$A\cos(-\omega t) = A\cos(\omega t)$$

Complex Exponential



$$Z = Z_x + jZ_y \qquad j^2=-1$$
$$= re^{j\theta}$$

Cartesian → Polar

$$r = \sqrt{Z_x^2+Z_y^2}$$
$$\theta = \tan^{-1}\left(\frac{Z_y}{Z_x}\right)$$

Polar → Cartesian

$$Z_x = r\cos\theta$$
$$Z_y = r\sin\theta$$

In the case when $r=1$, we have

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$ Euler Relationship

$e = 2.17 \ldots$

$e^{1.2} = (2.17 \ldots)^{1.2}$

$e^{j1.2} = ??$

Talyor series Representation

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{2 \cdot 3}x^3 + \frac{1}{2 \cdot 3 \cdot 4}x^4 + \cdots$$

$$e^{j\theta} \triangleq 1 + (j\theta) + \frac{1}{2}(j\theta)^2 + \frac{1}{2 \cdot 3}(j\theta)^3 + \frac{1}{2 \cdot 3 \cdot 4}(j\theta)^4 + \cdots$$

$$= 1 + j\theta - \frac{1}{2}\theta^2 - \frac{1}{2 \cdot 3}j\theta^3 + \frac{1}{2 \cdot 3 \cdot 4}\theta^4 + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5}j\theta^5 + \cdots$$

$$= \left(1 - \frac{1}{2}\theta^2 + \frac{1}{2 \cdot 3 \cdot 4}\theta^4 - \cdots\right) + j\left(\theta - \frac{1}{2 \cdot 3}\theta^3 + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5}\theta^5 - \cdots\right)$$

Talyor series

$$= \cos\theta + j\sin\theta$$

Taylor series

$$x(t) = Ae^{j\theta t} = A\cos\theta t + jA\sin\theta t.$$