Outline:
- Complex exponential series
- Frequency spectrum (magnitude & phase)
- DT periodic sequences
  1. Sampling Frequency
  2. Sampling CT periodic sequence: Is it periodic in DT? If yes, what is the period?
  3. Fundamental frequency - Nyquist Frequency Range

1. Thurs 5 pm (Not mandatory) “Here” - JAM SECTION
2. Drop problem 6 from HW 2 (last problem)

If \( x(t) \) is periodic with frequency \( \omega \)

\[
x(t) = \sum_{n=-\infty}^{\infty} a_n \cos(n\omega t + \phi_n)
\]

\[
= \sum_{n=-\infty}^{\infty} a_n \sin(n\omega t + \phi_n)
\]

\[
= \sum_{n=-\infty}^{\infty} b_n \sin(n\omega t) + \sum_{n=0}^{\infty} c_n \cos(n\omega t)
\]

\[
\Rightarrow = \sum_{n=-\infty}^{\infty} S_n e^{jn\omega t}
\]

To do this: use \( \sin(n\omega t) = \frac{1}{2j}[e^{jn\omega t} - e^{-jn\omega t}] \) negative frequencies

\( \cos(n\omega t) = \frac{1}{2}[e^{jn\omega t} + e^{-jn\omega t}] \)

and replace sine & cosine in (*) and group all the terms and obtain (A)
\[ x(t) = \sum_{n=-\infty}^{\infty} S_n e^{jnw_0 t} \]  

"Fourier Series"

\( \{S_n : -\infty < n < \infty\}\) completely specify the periodic signal \(x(t)\) with fundamental freq \(w_0\).

→ Write as a Frequency Spectrum of \(x(t)\)

defined as follows

\[ X(w) = S_0 + S_{-1} e^{j2w_0} + S_{-2} e^{j4w_0} + \ldots + S_{-1} e^{j(w_0)} + S_0 e^{j0} + S_{+1} e^{j(w_0)} + S_{+2} e^{j(2w_0)} + \ldots \]

free variable just like \(t\)

\[ \begin{array}{c}
S_{-2} \\
S_{-1} \\
S_0 \\
S_1 \\
S_2 \\
S_{+2} \\
\ldots
\end{array} \]

\(-2w_0\) \(-w_0\) \(0\) \(w_0\) \(2w_0\) \(\rightarrow w\)

Box

Graphical EQ

Low

High Freq

The line graph is not quite correct because \(S_n\) is complex! Two graphs:

Magnitude \(|S_n|\)

Phase \(\angle S_n\)
If non-periodic, still have a spectrum but the spectrum is continuous

\[ |X(\omega)| \]

\[ \omega \]

\[ X(\omega) \]

**DT Periodic Sequence**

\[ \text{CT signal} \]

\[ \leftrightarrow \]

**Sampling Period** = \( T \) seconds

**Sampling Frequency** = \( f_s = \frac{1}{T} \) (in Hz)

\[ = \omega_s = 2\pi f_s = \frac{2\pi}{T} \text{ rad s}^{-1} \]

- independent of the signal

What is a periodic DT sequence?

\[ x[n] = x[n+N] \quad \text{for all } n \]

\[ x[n] \]

\[ N = 4 \]

picks the smallest \( T = 1 \)

\[ n \]

\[ \{0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots \} \]
How do we get a DT periodic sequence?

\[ x(t) = \sin t \quad \text{Let's } T = 1 \]

Is \( x[n] = \sin(nT) = \sin(n) \) a periodic signal?

In other words, is there an \( N \) (integer) such that

\[ \sin(n) = \sin(n+N) \quad \text{for all } n \]

\[ = \sin(n) \cos(N) + \cos(n) \sin(N) \]

\[ \Rightarrow \cos(N) = 1 \quad \text{No integer will satisfy this} \]

\[ \Rightarrow \sin(N) = 0 \]

\( \Rightarrow x[nT] \) is not periodic !!!!!!

The problem here is that sampling period \( T=1 \)

is cannot divide \( P=2T \), which is the period of \( x(t) \)

\[ x(t) = \sin(wt) \]

Sampling at \( T \), we can a DT sequence \( x[n] \)
$x[n] = \sin (w n T)$

For $x[n]$ to be periodic, we must have

$x[n] = x[n+N]$ for some integer $N$

$\sin(w n T) = \sin[(w(n+N)T)]$

$= \sin(w n T + w N T)$

For this equality to hold, we must have

$w n T = k (2\pi)$ \hspace{1cm} $k$ is an integer.

\[
N = \frac{2\pi k}{w}
\]

Example: $w=1$, $T=1$,

\[
\frac{2\pi k}{w T} = \frac{2\pi k}{1 \cdot 1} = 2\pi k \neq N
\]

no matter what $k$ is

Example, $w=1$, $T=\pi/2$ \hspace{1cm} Change sampling period

\[
\frac{2\pi k}{w T} = \frac{2\pi k}{1 \cdot \pi/2} = 4k
\]

$k=1 \Rightarrow N=4 \cdot 1 = 4 \checkmark \hspace{1cm} x[n]$