Lecture 7

Note Title 9/7/2007

Outline : - complex exponential series
- frequency spectrum (magnitude & phase
- DT periodic sequences
① Sampling Frequency
② Sampling CT periodic sequence : Is it periodic in DT ? If yes, what is the period?
③ Fundamental frequency - Nyquist Frequency Range

① Thurs 5pm (Not mandatory) "Here" - JAM SECTION
② Drop Problem 6 (last problem) from HW2

If $x(t)$ is periodic with frequency ω

$$x(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t + \theta_n)$$

$$= \sum_{n=0}^{\infty} a_n \sin(n\omega t + \phi_n)$$

$$= \sum_{n=0}^{\infty} b_n \sin(n\omega t) + \sum_{n=0}^{\infty} c_n \cos(n\omega t) \qquad (*)$$

$$\rightarrow \quad = \sum_{n=-\infty}^{\infty} s_n e^{jn\omega t} \qquad (\Delta)$$

To do this : use $\sin(n\omega t) = \frac{1}{2j}[e^{jn\omega t} - e^{-jn\omega t}]$ negative frequencies

$$\cos(n\omega t) = \frac{1}{2}[e^{jn\omega t} + e^{-jn\omega t}]$$

and replase sine & cosine in (*) and group all the terms and obtain (Δ)

$$x(t) = \sum_{n=-\infty}^{\infty} S_n e^{+jn\omega_0 t}$$ "Fourier Series"

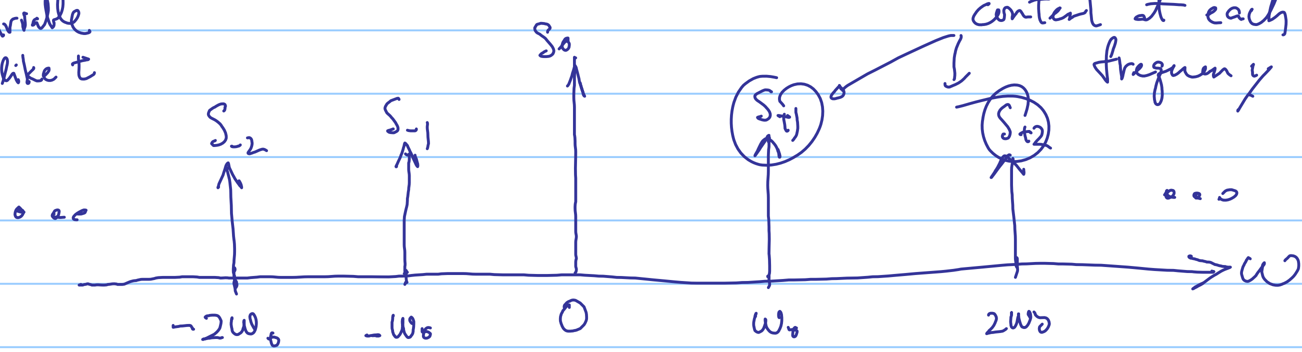
$\{S_n : -\infty < n < \infty\}$ completely specify the periodic signal $x(t)$ with fundamental freq ω_0

→ Write as a Frequency Spectrum of $x(t)$ defined as follows

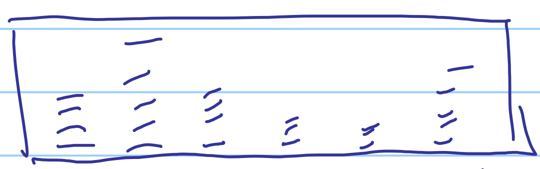
$$X(\omega) = \ldots + S_{-2}\delta(\omega + 2\omega_0) + S_{-1}\delta(\omega+\omega_0) + S_0\delta(\omega) + S_{+1}\delta(\omega - \omega_0) + S_{+2}\delta(\omega - 2\omega_0) + \ldots$$

free variable just like t

content at each frequency



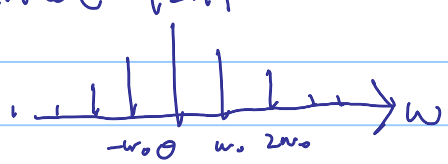
Bombox Graphic EQ



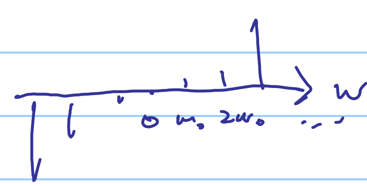
The above graph is not quite correct because S_n is Complex !!

Two graphs :

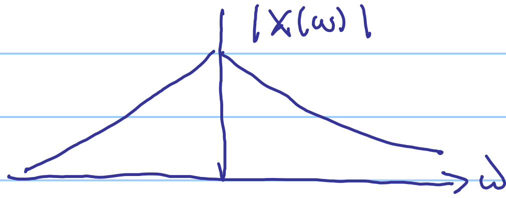
Magnitude $|S_n|$

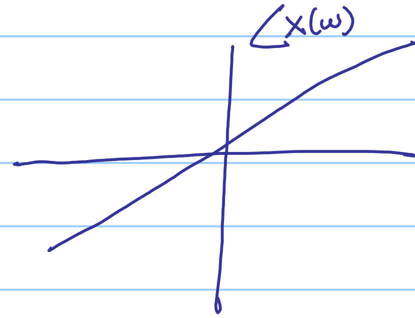


Phase $\angle S_n$

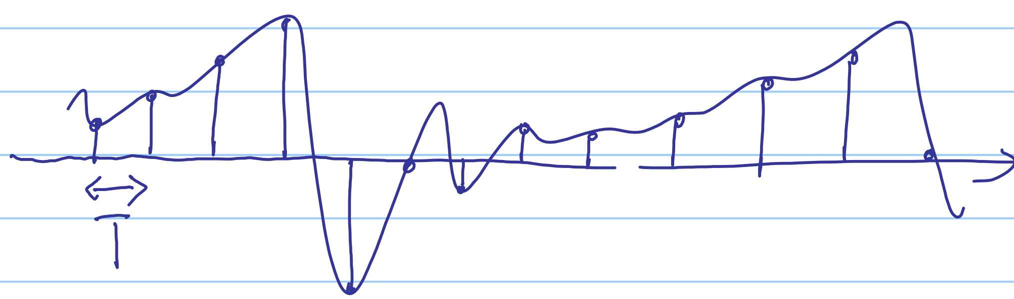


If non-periodic, still have a spectrum but the spectrum is continuous





DT Periodic Sequence



Sampling Period = T seconds

Sampling Frequency = $f_s = \frac{1}{T}$ (in Hz)

$= \omega_s = 2\pi f_s = \frac{2\pi}{T}$ rad s^{-1}

– independent of the CT signal

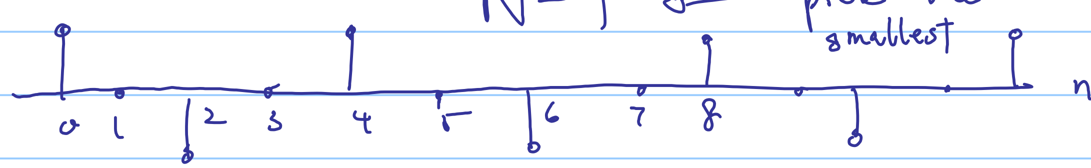
What is a periodic DT sequence?

$$x[n] = x[n+N] \quad \text{for all } n$$

$x[n]$

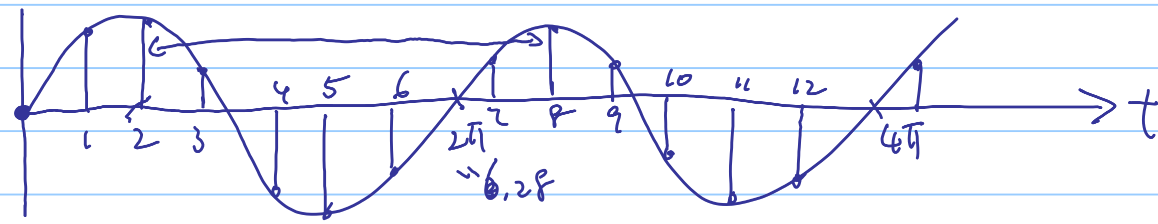
$N = 4$ ← pick the smallest (8, 12, 16 ...)

$T = 1$



How do we get a DT periodic sequence?

$x(t) = \sin t$ Lets $T = 1$



Is $x[n] = \sin(nT) = \sin(n)$ a periodic signal?

In other words, is there a N (integer) such that

$$\sin(n) = \sin(n+N) \quad \text{for all } n$$
$$= \sin(n)\cos(N) + \cos(n)\sin(N)$$

$$\Rightarrow \left.\begin{array}{l} \cos(N) = 1 \\ \sin(N) = 0 \end{array}\right\} \text{No integer will satisfy this}$$

$\Rightarrow x(nT)$ is not periodic !!!!

The problem here is that sampling period $T=1$ ~~is~~ cannot divide $P = 2\pi$, which is the period of $x(t)$

$x(t) = \sin(\omega t)$

Sampling at T, we can a DT sequence $x[n]$

$x[n] = \sin(\omega n T)$

For $x[n]$ to be periodic, we must have

$$x[n] = x[n+N] \quad \text{for some integer } N$$

$$\sin(\omega n T) = \sin(\omega(n+N)T)$$
$$= \sin(\omega n T + \omega N T)$$

For this equality to hold, we must have

$$\omega N T = k(2\pi) \qquad k = \text{an integer.}$$

$$\boxed{N = \frac{2\pi k}{\omega T}}$$

Example: $\omega = 1$, $T = 1$,

$$\frac{2\pi k}{\omega T} = \frac{2\pi k}{1 \cdot 1} = 2\pi k \neq N$$

no matter what k is

Example, $\omega = 1$, $T = \pi/2$ change sampling period

$$\frac{2\pi k}{\omega T} = \frac{2\pi k}{1 \cdot \pi/2} = 4k$$

$k = 1 \Rightarrow N = 4 \cdot 1 = 4$ period of $x[n]$