Lecture 9

Note Title 9/12/2007

Outline
- fundamental frequency of DT sinusoids
- Nyquist sampling & DEMO
- Systems : memory, casusality, linear, etc.

Given $x[n] = \sin(\omega_0 n T)$ assume periodic

① there are infinitely many ω that describe the same $x[n]$, i.e

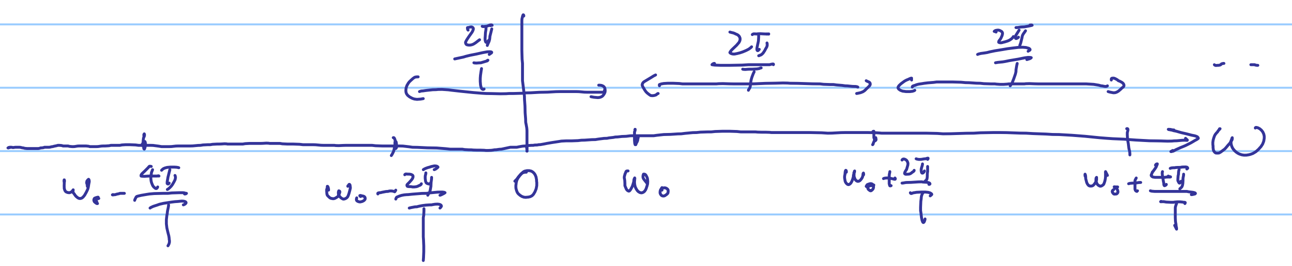
$$x[n] = \sin(\omega n T)$$

ω ← infinitely many of them.

$x[n]$ can be the sampled version (sampling period $= T$) of infinitely many CT $\sin(\omega t)$

② All these ω's differ by an integral # of $\frac{2\pi}{T}$ ← sampling period

$$\boxed{\omega = \omega_0 + k\frac{2\pi}{T}} \qquad (\triangle)$$

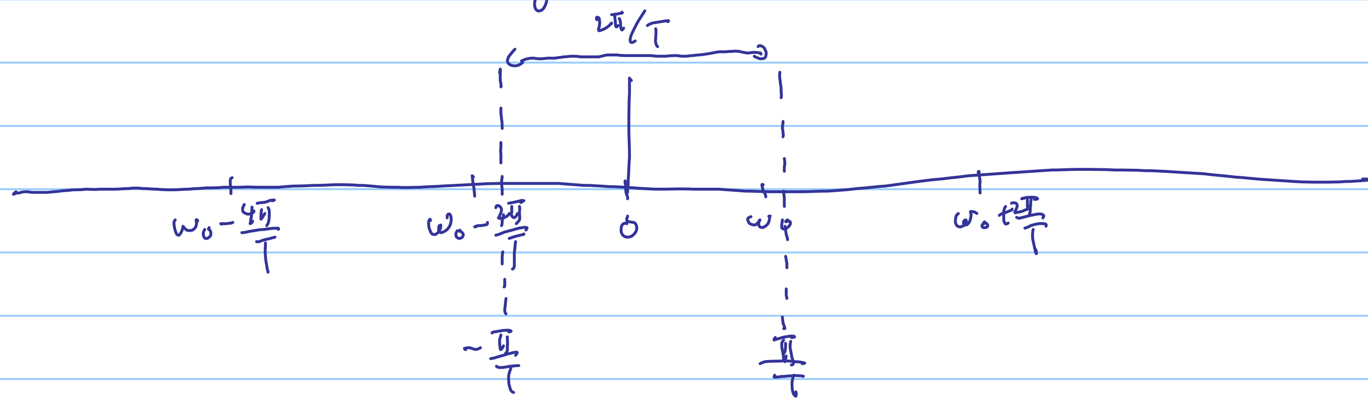


③ FUNDEMENTAL FREQUENCY of $x[n]$, $\tilde{\omega}$
= ω with the smallest magnitude (absolute value) that satisfies $(\triangle)$ relationship

You can find $\tilde{\omega}$ by finding the k such that

$$\omega = \omega_0 + k\frac{2\pi}{T}$$

is closest to the origin.

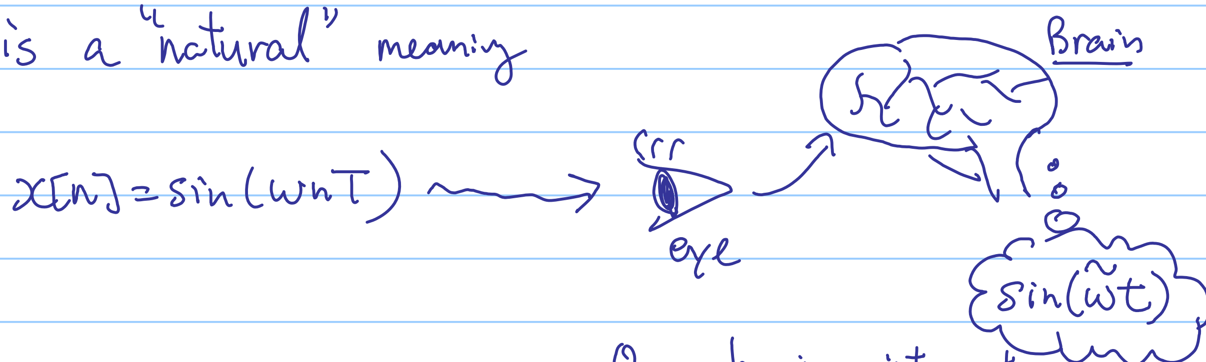


$\Rightarrow$ You can always find $\tilde{\omega}$ to be the pt within the interval $\left[-\frac{\pi}{T}, \frac{\pi}{T}\right)$ $\Rightarrow$ Nyquist Frequency Range.

Why do we care about $\tilde{\omega}$?

① $\tilde{\omega}$ is unique to a $x[n] = \sin(\omega n T)$

② It is a "natural" meaning



Our brain interprets this using the fundamental frequency.

$f_s = 12\,Hz \longrightarrow T = \frac{1}{12}s \longrightarrow$ NFR : $[-\frac{\pi}{T}, \frac{\pi}{T})$ in rad/s

$f = \frac{\omega}{2\pi}$ NFR : $[-\frac{1}{2T}, \frac{1}{T})$ Hz

$f_s = 12\,Hz \longrightarrow$ NFR : $[-6, 6)$ Hz

10.5 Hz is beyond $[-6, 6]$ Hz

$\downarrow -\frac{2\pi}{T}$ (rad/s) or -12 Hz

$10.5 - 12\,Hz = -1.5\,Hz \leftarrow$ fundamental frequency

appears to rotate backwards

Appearance of the "low" fundamental frequency when sampling a high frequency CT signal

$\Rightarrow$ ALIASING

In other words, you are given a CT signal

$$x(t) = \sin(\omega t)$$

What should be the sampling period T such that the sampled version

$$x[n] = \sin(\omega n T)$$

will have the fundamental frequency $\tilde{\omega}$ same as ω?

desired freq

NFR : $[-\frac{\pi}{T}, \frac{\pi}{T})$ Want $\omega < \frac{\pi}{T} \Rightarrow T < \frac{\pi}{\omega}$

Put it in regular frequency

$\omega = 2\pi f$

$\frac{\pi}{\omega} = \frac{1}{2} f$ $\Rightarrow$ $T < \frac{1}{2} f$

$\Rightarrow$ $\boxed{f_s > 2f}$

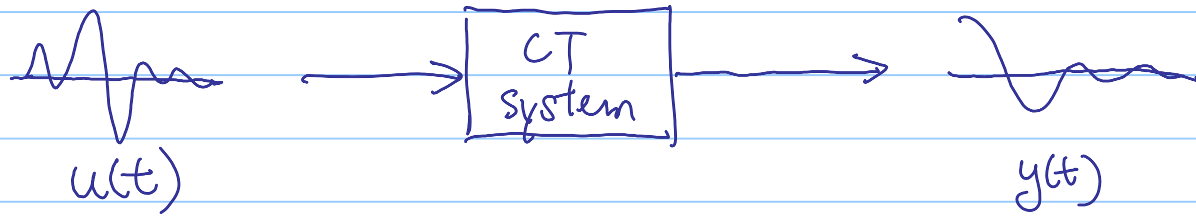
sampling frequency

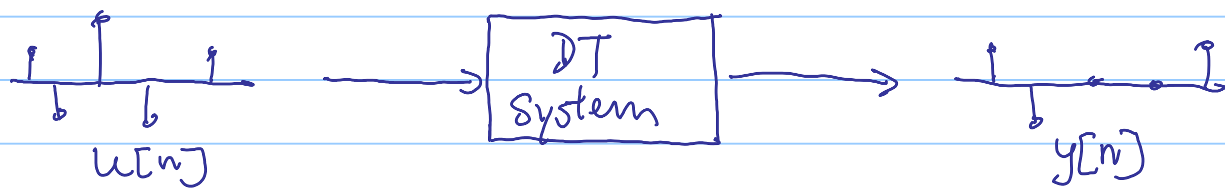
frequency of sinusoid
CT

$\Rightarrow$ If you want to perceive f, you need to sample at least twice the frequency.

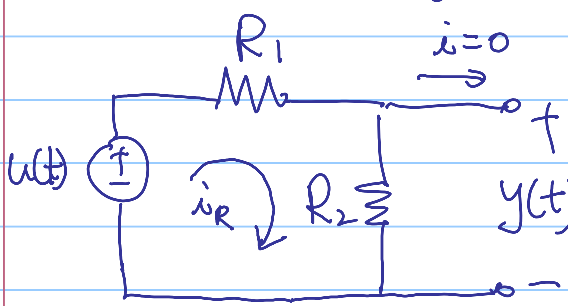
$2f$ - Nyquist Sampling frequency.

System (Ch 2)





Example 1 Voltage Divider



Assume we are not drawing any current to the output

$$\begin{cases} u(t) = i_R(t) R_1 + \cancel{y(t)}\, i_R(t) R_2 \\ \quad\;\; = i_R(t)(R_1 + R_2) \\ y(t) = R_2\, i_R(t) \end{cases}$$

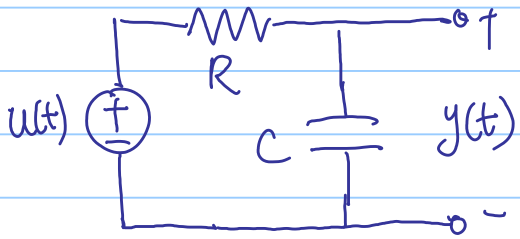
$$\Rightarrow \quad u(t) = \frac{y(t)}{R_2}(R_1 + R_2)$$

$$y(t) = \frac{R_2}{R_1 + R_2} u(t)$$

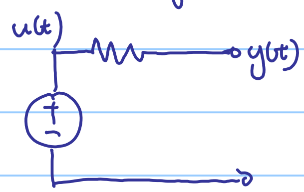
Output at time t depends only on the input at time t

$\Rightarrow$ Memoryless system

Example 2 Low-pass filter.

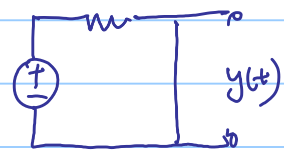


at Low freq



$y(t) \approx u(t)$

High freq



$y(t) \approx 0$