EE422G Homework #13 (12 points)

Due April 24, 2007

1. (3 points) In this problem, you are asked to explore an important application of FFT: efficient computation of convolution. The impulse response of a system is given by

   $h(nT) = (0.9)^n \qquad n = 0,1,2,\ldots,31$

   and the input to the system is given by

   $x(nT) = 3\sin(\tfrac{1}{4}\pi n) \qquad n = 0,1,2,\ldots,31$

   You are asked to use the `fft` and `ifft` commands in Matlab to compute and plot the output y(nT) = h(nT) * x(nT). In addition, please answer the following questions:
   a. What is the minimum value of N in the N-point DFT (FFT) needed to compute this linear convolution?
   b. If the impulse response has N non-zero samples and the input has M non-zero samples, how many arithmetic operations (additions and multiplications) are needed to compute the linear operation from its definition, i.e.
   $$y(nT) = \sum_{m=0}^{N-1} h(mT)x(nT-mT) \quad for\ n = 0,1,\ldots,M-1?$$
   c. Repeat b) but using the FFT method.

2. (4 points) During lecture, we discussed the decimation-in-time method of implementing FFT in which we partition the sequence into even and odd time samples. There is an alternative approach called *decimation-in-frequency* which we are going to explore in this problem. Recall that a 8-point DFT is given by
   $$X(k) = \sum_{n=0}^{7} x(nT)W_8^{nk} \quad k = 0,1,\ldots,7 \qquad (1)$$
   where $W_N^{nk} = \exp\left(\frac{j2\pi nk}{N}\right)$.
   a. Show that
   $$X(k) = \sum_{n=0}^{3} \left[x(n) + (-1)^k x(4+n)\right] W_8^{nk} \qquad (2)$$
   b. For even k, show that equation (2) is equivalent to the 4-point DFT of the sequence $y(n) = x(n) + x(4+n)$
   c. For odd k, show that equation (2) is equivalent to the 4-point DFT of the sequence $y(n) = \left[x(n) - x(4+n)\right] W_8^{n}$
   d. Draw the butterfly network of the 8-point DFT in terms of the preprocessing stage followed by two 4-point DFTs.

3. (3 points) Starting with the analog first-order low pass filter, H(s) = 5/(s+4).
   a. Find the response due to the following inputs: $\delta(t)$, u(t) and $e^{-4t}u(t)$.
   b. For the low pass filter filter in part a), design a digital filter H(z) with T_s=10ms using i) Impulse invariant, ii) step invariant and iii) bilinear transformation.
   c. Using T=10ms and Matlab's filter() command, plot the response of each of your digital filter designs to the following discrete inputs: $(1/T)\delta(nT)$, u(nT) and $e^{-4nT}u(nT)$. Compare with the analog outputs from part a.

4. (2 points) Design a digital filter for the analog prototype $H(s) = \dfrac{1}{(s+5)(s+10)}$ using both the impulse-invariant and the bilinear methods. The sample period of the digital filter is 0.1 second. Plot the amplitude and phase responses of the analog filter as well as the two digital filters and comment on their differences.