

EE422G Homework #1 Solution

1. (4 pts) Determine which of the following systems are linear. Explain your answer:

(a) $\frac{dy(t)}{dt} + 3y(t) = 4x(t)$

Solution: Yes, it is linear. The reason is that given two pairs of input-output signals $x_1(t), y_1(t)$ and $x_2(t), y_2(t)$ that satisfy the differential equation, it is clear that $ax_1(t) + bx_2(t)$ and $ay_1(t) + by_2(t)$ are also a solution to the differential equation as shown below:

$$\begin{aligned} \frac{d}{dt}(ay_1(t) + by_2(t)) + 3(ay_1(t) + by_2(t)) &= a \left[\frac{d}{dt}y_1(t) + 3y_1(t) \right] + a \left[\frac{d}{dt}y_2(t) + 3y_2(t) \right] \\ &= ax_1(t) + bx_2(t) \end{aligned}$$

(b) The impulse response: $h(t) = \delta(t + 2)$

Solution:

Since the system has an impulse response, it is definitely linear.

(c) $y(t) = x^2(t) + 2\frac{dx(t)}{dt}$

Solution:

It is non-linear due to the second degree term – when using a similar test as part a), you will notice that the second degree term creates a cross term $x_1(t)x_2(t)$ that is not present in the differential equation.

(d) Solution:

The system is non-linear. This is a little bit tricky. As we know, for all linear systems, we can relate the input and output Fourier Transforms using the following relationship:

$$Y(j\omega) = X(j\omega)H(j\omega)$$

where $H(j\omega)$ is the Fourier Transform of the impulse response. Using this relationship, if $X(j\omega)$ is zero at a particular frequency, $Y(j\omega)$ must also be zero no matter what $H(j\omega)$. This is clearly not the case here as $X(j\omega)$ is zero except at ω_0 and $-\omega_0$.

2. (2 pts) Give the definition of Fourier transform and Fourier series. Does every signal have a Fourier transform? If yes, prove it. If no, give a counter-example.

Solution:

The Fourier transform of a sequence $x[n]$ is defined as:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

The discrete Fourier series analysis-synthesis pair is expressed as:

Analysis equation:

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]e^{-\frac{j2\pi kn}{N}}$$

Synthesis equation:

$$\tilde{x}[k] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{X}[n] e^{j2\pi kn/N}$$

Fourier Transform does not exist for signals whose Laplace Transforms have poles on the open right half plane. An example of such signal is $x(t) = e^t u(t)$.

3. (2 pts) (THIS PROBLEM WILL NOT BE GRADED) Get the Laplace transform for the following transform and the corresponding region of convergence (ROC):

(a) $x(t) = \sin(\omega t)u(t)$

Solution:

$$\sin(\omega t) = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

$$e^{-at}u(t) \Leftrightarrow \frac{1}{s+a}$$

By the linearity property, the sum of these terms corresponds to the sum of their Laplace transforms. Therefore,

$$\mathcal{L}[\sin(\omega t)u(t)] = \frac{1}{2j} \left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right] = \frac{\omega}{s^2 + \omega^2}$$

for $\sigma > 0$

(b) $x(t) = \delta(t - t_0), t_0 > 0$

Solution:

$$X(s) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-st} dt = e^{-st_0}$$

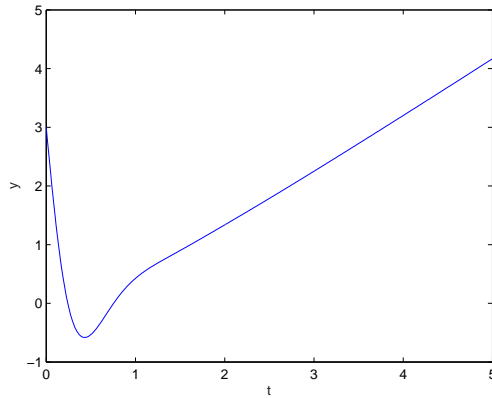
for all s

4. (2 pts) Use Matlab to plot y versus t with $t = 0 : 0.01 : 5$ and y defined as:

$$y = 3e^{-4t} \cos(5t) - 2e^{-3t} \sin(2t) + \frac{t^2}{t+1}$$

Solution:

```
t=0:0.01:5;
y=3*exp(-4*t).*cos(5.*t)-2*exp(-3*t).*sin(2*t)+t.^2./(t+1);
plot(t,y)
xlabel('t')
ylabel('y')
```



5. (2 pts) Given two polynomials:

$$f(x) = x^5 - 3x^4 + 5x^2 + 7x + 9$$

and

$$g(x) = 2x^6 - 8x^4 + 4x^2 + 10x + 12$$

Find all the solutions to equations:

$$f(x)g(x) = 0$$

Hint: Run help on Matlab commands “roots” and “conv”.

Solution:

```
f = [1 -3 0 5 7 9];
g = [2 0 -8 0 4 10 12];
fg = conv(f,g)
roots(fg)
```

```
ans =
2.4186 + 1.0712i
2.4186 - 1.0712i
1.6760 + 0.4922i
1.6760 - 0.4922i
-1.9304
-0.2108 + 0.9870i
-0.2108 - 0.9870i
-0.3370 + 0.9961i
-0.3370 - 0.9961i
-1.1633
-1.0000
```