EE422G Homework #3 (12 points)
Due February 1, 2007

1. (2 points)
   a. Prove the Integration Theorem
   
   b. Find the integral-differential equation that corresponds to the following equation in s domain.
   
   \[s^2 Y(s) + 2sY(s) = s^3 X(s) + \frac{X(s)}{s} - e^{-s/s}\]

2. (2 points) Solve for x(t) and y(t) which are governed by the following system of differential equations:

   \[
   \frac{dx(t)}{dt} + 3x(t) + 2y(t) = u(t) \\
   \frac{dy(t)}{dt} - x(t) = 0
   \]

   Boundary conditions: \(x(0) = y(0) = 0\)

3. (2 points) Obtain the Laplace transform of the triangular signal \(x(t) = \Lambda(t-1)\) by using the differentiation theorem, the time-delay theorem, and expressing \(\frac{dx}{dt}\) in terms of unit steps.

4. (4 points) Find the initial and final values, if exists, of the signals with Laplace transforms given below:
   
   a. \[
   s^3 + s^2 + 9s + 9
   \]
   
   b. \[
   s^2 + 5s + 7 \\
   s^2 + 3s + 2
   \]

5. (4 points) Obtain the inverse Laplace transform of
   
   a. \[
   X(s) = \frac{7s^2 + 15s + 10}{(s+1)^2(s+3)}
   \]
   
   b. \[
   X(s) = \frac{s^4 + 8s^2 + s + 17}{(s^2 + 4)^2(s+1)}
   \]
6. (4 points) Given \( Y_1(s) = \frac{s^3 + 6s^2 + 11s + 6}{(s^2 + 4)^2} \) and \( Y_2(s) = \frac{s^3 + 6s^2 + 11s + 6}{s^2 + 4s + 4} \)

a. Use MATLAB to find \( x(t) = y_1(t) \ast y_2(t) \)
b. Use MATLAB’s residue function to find the partial fraction expansions for \( Y_1(s) \) and \( Y_2(s) \)