EE422G Homework #6 (11 points)

Due February 20, 2007 (NEXT TUESDAY! NOT THURSDAY.)

1. (3 points) Without computing the inverse Laplace transform, determine the type(s) of stability exhibited by the following system:
   a. $H(s) = \dfrac{s-2}{s^2+7s+12}$
   b. $H(s) = \dfrac{s+3}{s^2-2s+2}$
   c. $H(s) = \dfrac{s^2-1}{s^4-4s^3+8s^2-8s+4}$

2. (4 points) You have seen the dramatic collapse of Tacoma Narrows Bridge during lecture. In this problem, we will analyze the cause of the collapse. The left figure below schematically shows the vertical deflection y(t) and the horizontal deflection angle x(t) of the bridge. Of particular importance is the horizontal deflection angle x(t) which is governed by the following differential equation:

$$\frac{d^2}{dt^2}x(t) + c\frac{d}{dt}x(t) + kx(t) = f(t)$$

   where c > 0 is the coefficient of viscous damping divided by the mass, k = 1 is the Hooke's law spring constant of the cables divided by the mass, and f(t) is the acceleration of the bridge due to the wind.
   a. In the original design of the bridge (the middle figure), the parameter c is found to be very close to 0. Assuming c = 0 and modeling the bridge as H(s) = X(s)/F(s), what type of stability (BIBO, asymptotic, marginal or unstable) does the system exhibit? Justify your answer.
   b. The bridge was later rebuilt with a new design in which engineers replaced the stiffening-plate girders with web trusses as shown in the right figure. This increases the value of c to around two. Comment on the stability of this new design.

Figure 1: Tacoma Narrow Bridge

3. (4 points) (Routh Array) Use Routh Array for the following problems:

(a) Does the following system have any poles on the open right half plane?

$$H(s) = \frac{1}{s^4 + s^3 + 12s^2 + 12s + 36}$$

(b) Show that a necessary condition for the following system is that -5 ≤ K ≤72.

$$H(s) = \frac{1}{s^3 + 7s^2 + 11s + 5 + K}$$