EE422G Homework #6 Solution

1. (3 points) Without computing the inverse Laplace transform, determine the type(s) of stability exhibited by the following systems:

   Solution:

   (a) $H(s) = \dfrac{s-2}{s^2+7s+12}$ is BIBO and thus asymptotically and marginally stable as well. The reason is that
   (a) $H(s)$ is proper as degree $(s-2) <$ degree$(s^2+7s+12)$
   (b) The poles are at -3 and -4 (as $s^2+7s+12 = (s+3)(s+4)$) and they are all on the open left half plane.

   (b) $H(s) = \dfrac{s+3}{s^2-2s+2}$ is unstable because the poles are at $1+j$ and $1-j$ which are on the right half plane.

   (c) $H(s) = \dfrac{s^2-1}{s^4-4s^3+8s^2-8s+4}$ is unstable because the poles are at $1+j$ (multiplicity=2) and $1-j$ (multiplicity=2) which are on the right half plane.

2. (4 points) (Tacoma Narrows Bridge) You have seen the dramatic collapse of Tacoma Narrows Bridge during lecture. In this problem, we will analyze the cause of the collapse. The left figure below schematically shows the vertical deflection $y(t)$ and the horizontal deflection angle $x(t)$ of the bridge. Of particular importance is the horizontal deflection angle $x(t)$ which is governed by the following differential equation:

$$\frac{d^2}{dt^2}x(t) + c\frac{d}{dt}x(t) + kx = f(t)$$

   where $c > 0$ is the coefficient of viscous damping divided by the mass, $k = 1$ is the Hooke's law spring constant of the cables divided by the mass, and $f(t)$ is the acceleration of the bridge due to the wind.

   (a) In the original design of the bridge (the middle figure), the parameter c is found to be very close to 0. Assuming $c = 0$ and modeling the bridge as $H(s) = X(s)/F(s)$, is it a BIBO system? Justify your answer.

   Solution:
   We use the Laplace transform and get:

$$s^2X(s) - sx(0^-) - x^{(1)}(0^-) + csX(s) - cx(0^-) + kX(s) = F(s) \qquad (1)$$

   We know $c = 0, k = 1$. To compute the transfer function, we further set all the initial conditions to zero:

$$s^2X(s) + X(s) = F(s)$$

$$X(s) = \frac{F(s)}{s^2+1}$$

The transfer function $H(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2+1}$. The poles are on the imaginary axis: $(0, j), (0, -j)$, so this system is not BIBO. It is a Marginally Stable system.

(b) The bridget was later rebuilt with a new design in which engineers replaced the stiffening-plate girders with web trusses as shown in the right figure. This increases the value of c to around two. Comment on the validity of this new design.

Solution:
In this case, we put $c = 2$ into Eq. 1 and get:

$$s^2X(s) + 2sX(s) + X(s) = F(s)$$

The transfer function $H(s) = \frac{X(s)}{F(s)} = \frac{1}{(s+1)^2}$. The poles are on the open left half plane and it is BIBO.