

EE422G Homework #7 (14 points)

Due March 1, 2007

1. (4 points)

- a. Use Routh array to show that the following transfer function has two poles on the open right half plane. (2 points)

$$H(s) = \frac{1}{s^4 + s^3 + 12s^2 + 12s + 36}$$

s^4	1	12	36
s^3	1	12	
s^2	$\theta - \epsilon$	36	
s	$(12\epsilon - 36)/\epsilon$		
s	36		

For small $\epsilon > 0$, $(12\epsilon - 36)/\epsilon = 12 - 36/\epsilon < 0$ for small ϵ . We have two sign changes and thus the transfer function has two poles on the open right half plane.

- b. Show that if the following transfer function does not have any open r.h.p. poles, K must obey $-5 \leq K \leq 72$ (make sure you show that K can actually be -5 or 72).

$$H(s) = \frac{1}{s^3 + 7s^2 + 11s + 5 + K} \quad (4 \text{ points})$$

s^3	1	11
s^2	7	$5 + K$
s	$(72 - K)/7$	
1	$5 + K$	

If we want the system to have no poles on the right half plane, there could not be any sign changes in the first column. We need to consider two cases: case 1) all entries are positive; case 2) some entries are zero.

Case 1: All entries are positive. This implies

$$(72 - K)/7 > 0 \quad \text{AND} \quad 5 + K > 0$$

$$\Rightarrow K < 72 \quad \text{AND} \quad K > -5$$

$$\Rightarrow -5 < K < 72$$

Case 2: Some entries are zero

$$(72 - K)/7 = 0 \quad \Rightarrow \quad K = 72$$

In this case, the routh array becomes

s^3	1	11
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s^3	7	77	
s^2	7		← replaced by auxiliary polynomial
1	77		

⇒ No sign change ⇒ $K=72$ is acceptable

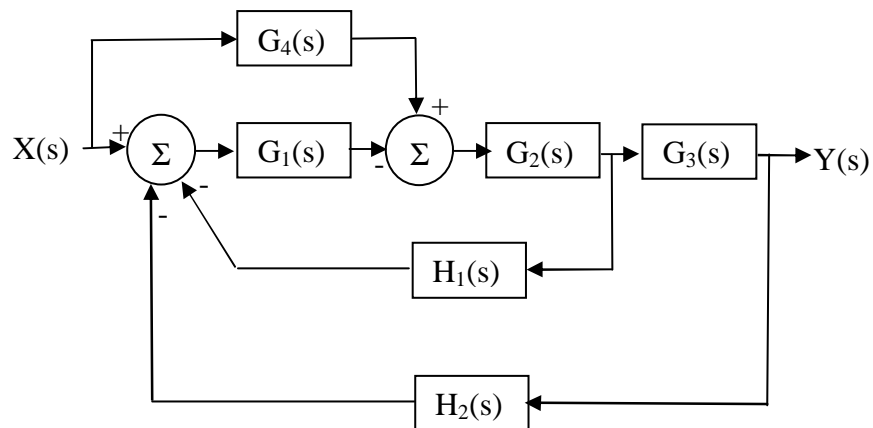
$5+K=0 \Rightarrow K=-5$
 The Routh array becomes

s^3	1	11	
s^2	7	0	
s^1	11		
1	11		← replaced by auxiliary polynomial

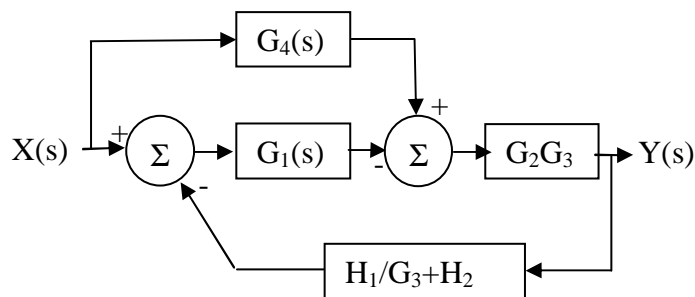
⇒ No sign change ⇒ $K=72$ is acceptable

Thus, the acceptable range is $-5 \leq K \leq 72$

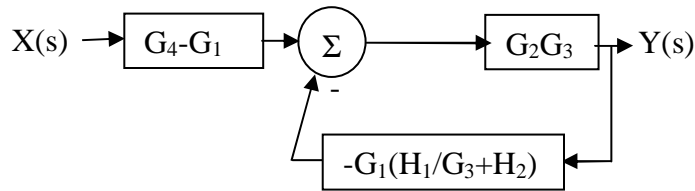
2. (2 points) Find the transfer function $Y(s)/X(s)$



There are many approaches to solve this problem. Here is one way:
 First move $G_3(s)$ back before the intersection so that the two feedback branches become parallel and I can combine them.



Then, move $-G_1$ back through the first summer so that I can combine the two summers together. Note that $-G_1$ and G_4 are in parallel so I combine them as well.



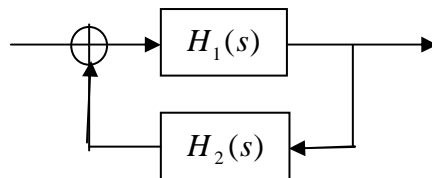
This is nothing but G_4-G_1 followed by a feedback loop. We can immediately write out the transfer function as:

$$H(s) = \frac{G_2(s)G_3(s)[G_4(s) - G_1(s)]}{1 - G_1(s)G_2(s)G_3(s)[\frac{H_1(s)}{G_3(s)} + H_2(s)]}$$

$$H(s) = \frac{G_2(s)G_3(s)[G_4(s) - G_1(s)]}{1 - G_1(s)G_2(s)[H_1(s) + G_3(s)H_2(s)]}$$

3. (4 points) Feedback System

- a. (2 points) Consider the negative feedback system below with $H_1(s) = \frac{-10}{s+1}$ and $H_2(s) = 2$. Check the stability of $H_1(s)$, $H_2(s)$ and the feedback system. Is it true that if all subsystems are stable, then the negative feedback system is also stable?



$H_1(s)$ is BIBO stable and $H_2(s)$ are asymptotically stable. The transfer function of the feedback system is

$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)} = \frac{-10}{s-19}$$

which is unstable. Thus it shows that a system can be unstable even if all the subsystems are stable.

- b. (2 points) Design the inverse systems for

$$H_1(s) = \frac{s+1}{s^2+2s+5} \text{ and } H_2(s) = \frac{s-1}{s^2+2s+5}$$

$H_1(s)$ has a zero at -1 which means that its inverse $G_1(s) = 1/H_1(s)$ has a pole at -1. Thus, $G_1(s)=1/H_1(s)$ is an asymptotically stable inverse for $H_1(s)$.

$H_2(s)$ has a zero at 1 which means that $1/H_2(s)$ is unstable. We can approximate its inverse by using a negative feedback system:

$$G_2(s) = \frac{A}{1 + AH_2(s)} = \frac{A(s^2 + 2s + 5)}{s^2 + 2s + 5 + As - A} = \frac{A(s^2 + 2s + 5)}{s^2 + (2 + A)s + (5 - A)}$$

To make this at least marginally stable, we need to restrict A such that none of its poles are in the open right half plane. Idea: use Routh array:

$$\begin{array}{ccc} s^2 & 1 & 5 - A \\ s & 2 + A & 0 \\ 1 & 5 - A & \end{array}$$

If $-2 < A < 5$, then the first column will all be positive and thus no open right-half plane.

If $A = -2$, we have

$$\begin{array}{ccc} s^2 & 1 & 7 \\ s & 1 & 0 \\ 1 & 7 & \end{array}$$

All entries in the first column are positive, so $A = -2$ is okay.

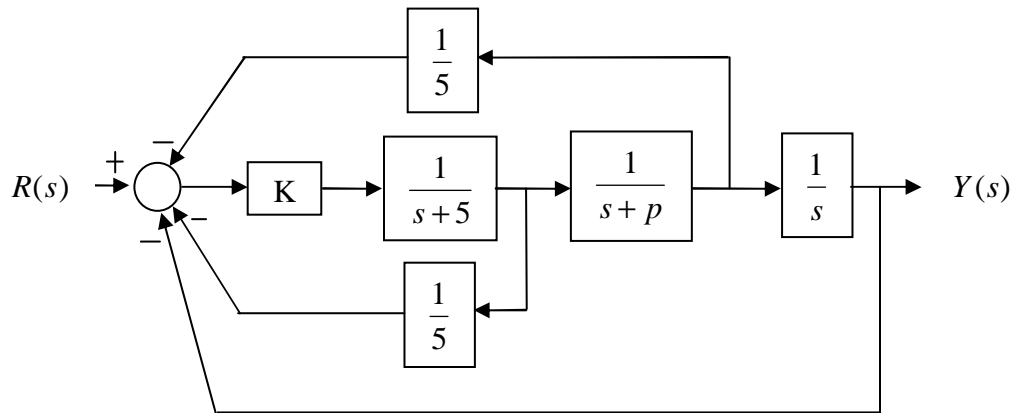
If $A = 5$, we have

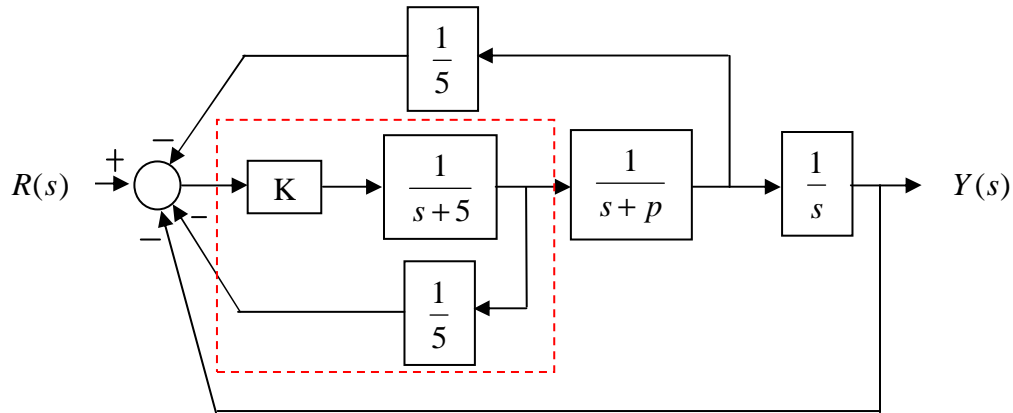
$$\begin{array}{ccc} s^2 & 1 & 0 \\ s & 7 & 0 \\ 1 & 7 & \end{array}$$

All entries in the first column are positive, so $A = 5$ is okay.

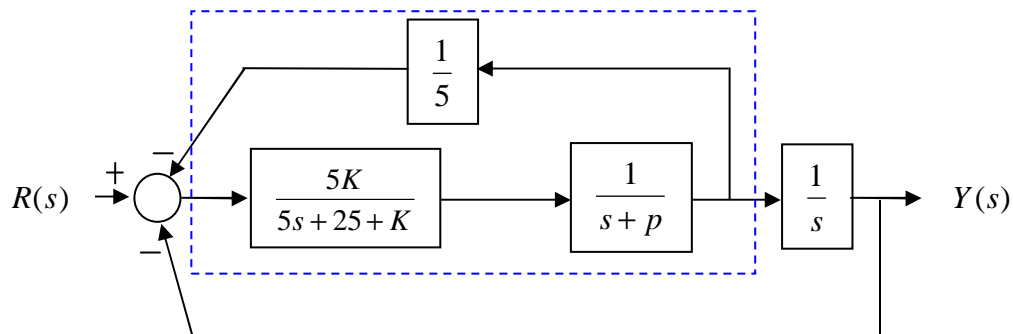
Thus, $G_2(s)$ will not have any poles in the open right half plane if $-2 \leq A \leq 5$.

4. (4 points) The control of the spark ignition of an automotive engine requires constant performance over a wide range of parameters. The control system is shown in the following figure, with a controller gain K to be selected. The parameter p is equal to 2 for many autos but can equal zero for those with high performance. Select a gain K that will result in a stable system for both values of p . You can ignore the case when the poles may lie on the imaginary axis.



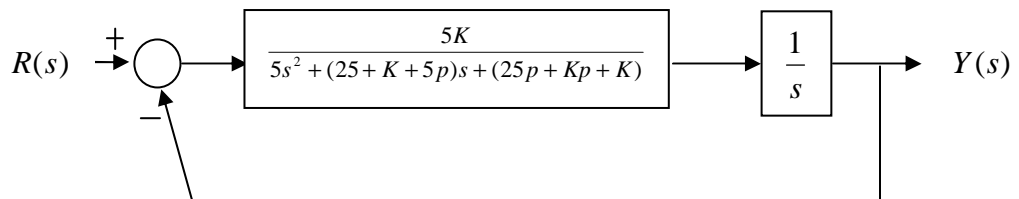


First notice that the blocks surrounded by the red box is a feedback system which can be replaced by open-loop system $\frac{K/(s+5)}{1+K/[5(s+5)]} = \frac{5K}{5s+25+K}$



Again, the blue box again is a feedback system:

$$\frac{5K/[(5s+25+K)(s+p)]}{1+K/[(5s+25+K)(s+p)]} = \frac{5K}{5s^2+(25+K+5p)s+(25p+Kp+K)}$$



Now, we encounter our last feedback loop:

$$\frac{5K/[5s^3+(25+K+5p)s^2+(25p+Kp+K)s]}{1+5K/[5s^3+(25+K+5p)s^2+(25p+Kp+K)s]} = \frac{5K}{5s^3+(25+K+5p)s^2+(25p+Kp+K)s+5K}$$

Setting up the Routh Array:

For $p=2$:

s^3	5	(50+3K)
s^2	(35+K)	5K
s	$(3K^2+80K+1750)/(35+K)$	
1	5K	

To guarantee not left half plane poles, we need to have

1. $35+K>0 \Rightarrow K>-35$

2. $(3K^2+80K+1750)/(35+K)>0 \Rightarrow K>-35$

As the numerator quadratic polynomial has no real roots, it is always larger than 0. Thus, all we need is the denominator polynomial to be larger than 0, which is the same condition as 1.

3. $5K>0 \Rightarrow K>0$

Combining all three, the condition is $K>0$.

For $p=0$:

s^3	5	K
s^2	(25+K)	5K
s	$K^2/(25+K)$	
1	5K	

To guarantee not left half plane poles, we need to have

1. $25+K>0 \Rightarrow K>-25$

2. $K^2/(25+K)>0 \Rightarrow K>-25$

3. $5K>0 \Rightarrow K>0$

Thus, as long as $K>0$, it should satisfy both cases for $p=0$ and $p=2$.