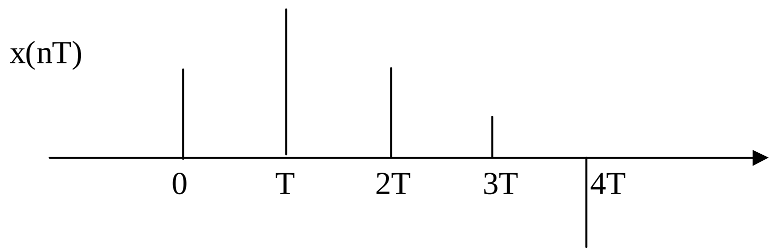
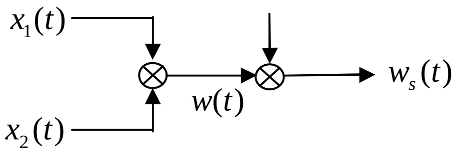
EE422G Homework #9 (15 points)

Due March 22, 2007

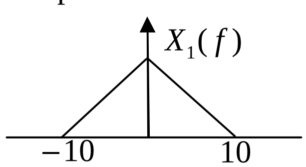
1. (3 points) Sampling and Reconstruction
   a) Given the following discrete-time input x(nT), sketch the outputs reconstructed continuous signal y(t) using Sample-And-Hold and Linear Interpolation.

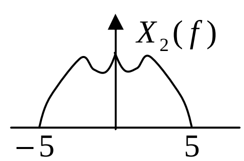


   b) In the following system, two continuous-time functions $x_1(t)$ and $x_2(t)$ are MULTIPLIED and the product $w(t)$ is sampled with sampling period T.



   If the spectrums of $x_1(t)$ and $x_2(t)$ are given as follows, determine the maximum sampling period T such that $w(t)$ is recoverable from $w_s(t)$ through the use of an ideal lowpass filter.





2. (2 points) An A/D converter has an input signal given by the Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

Show the signal-to-noise ratio at the output of the A/D converter is given by

$$SNR = \frac{12}{D^2} 2^{2n} \sum_{n=-\infty}^{\infty} \left| X_n \right|^2$$

where D is the dynamic range.

3. (4 points) An RC low-pass filter having transfer function

$$H(f) = \frac{1}{1 + j(f / f_3)}$$

where f_3 is the 3-dB frequency of the filter, is used for reconstruction of a signal from its samples. We can ignore the multiplying constant T which can be realized as a multiplier external to the filter. The signal $x(t)$ to be reconstructed is assumed to be a low-pass signal with bandwidth W. We wish to determine an appropriate sampling frequency so that the original spectrum $X(f)$ is passed by the filter with negligible error and that the translated spectra $X(f \pm f_s)$ were sufficiently attenuated.

a. Let $20\log|H(W)| = \delta_1$ dB and $20\log|H(f_s - W)| = \delta_2$ dB. We clearly wish δ_1 to correspond to negligible attenuation relative to the dc gain and δ_2 to a large attenuation. For fixed δ_1, δ_2 and signal bandwidth *W*, show that necessary sampling frequency, f_s, is

$$f_s = W\frac{\sqrt{10^{-0.1\delta_1}-1}+\sqrt{10^{-0.1\delta_2}-1}}{\sqrt{10^{-0.1\delta_1}-1}}$$

b. Use the result to determine the ratio f_s/W for

i. $\delta_1 =$-3 dB and $\delta_2 =$-30dB

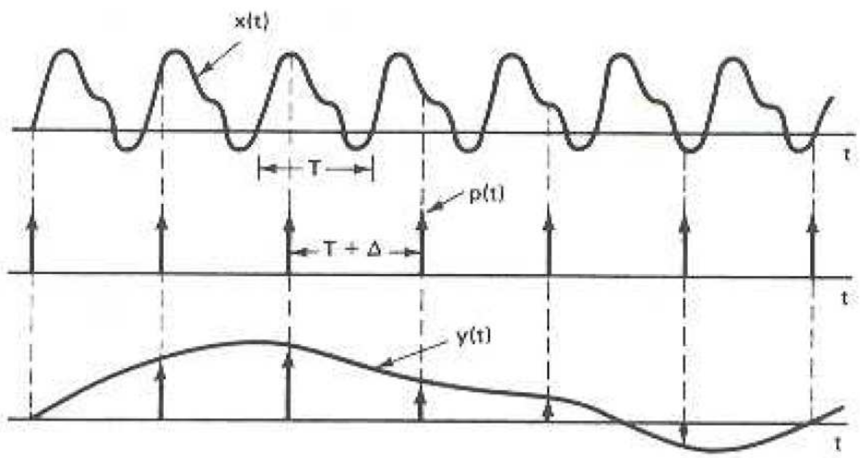
ii. $\delta_1 =$-1 dB and $\delta_2 =$-30dB

iii. $\delta_1 =$-1 dB and $\delta_2 =$-40dB

Comment on these results. How can large values of f_s/W be avoided?

The following two problems on sampling are quite challenging. They are meant to strengthen your understanding of the power and limitation of the Nyquist Sampling Theorem. Enjoy them and feel free to discuss with your peers and me.

4. (3 points) It is often necessary to display on an oscilloscope screen waveforms having very short time structures, for example, on the scale of thousandths of a nanosecond. Since the rise time of the fastest oscilloscope is longer than this, such display cannot be achieved directly. If however, the waveform is periodic, the desired result can be obtained indirectly using an instrument called a sampling oscilloscope. As shown in the figure below, the fast waveform x(t) is sampled once every period but at successively later points in successive periods. If the resulting impulse train is then passed through an appropriate low-pass interpolating filter the output, y(t), will be proportional to the original fast waveform slowed down or stretched out in time; i.e. y(t) is proportional to x(at) where a<1.



Assume that the input is a sinusoid x(t) = A+Bcos[(2π/T)t+θ]. Draw a block diagram of a system that impulse train samples x(t) via p(t) and then "immediately" reconstructs y(t) from the impulse train sampled signal using a low-pass filter. What is the maximum cutoff of the low-pass filter? Find the range of Δ so that y(t) is proportional to x(at) with a<1.

5. (3 points) In order to earn extra travel money for your spring break vacation, you signed up for a temporary job collecting data at the Astronomy Department between 1:00am and 6:00am. The job was to take the reading x(nT) from a radio telescope every half an hour (T=30) – thus you need to get 11 readings per night. As x(nT) does not change much from reading to reading, the job had become quite boring after the first two nights. On the third night, you had fallen asleep in your dorm room and did not get to the lab until 5:19 am. Hoping to somehow correct your mistakes, you frantically took down eleven readings everything 4 minutes starting at 5:20am.

   On your way back to your dorm, you kept thinking how you could report the values to the Astronomy Professor. All of a sudden, you remembered about the sync interpolation that you learned in EE422G:

$$x(t) = \sum_{n=0}^{\infty} x(nT)\operatorname{sinc}\left(\frac{t-nT}{T}\right)$$

   The Astronomy professor already convinced you that T=30 minutes is sufficient to produce a sampling rate higher than the Nyquist sampling frequency. Since your job were to take 11 readings, you conclude that you can reconstruct x(t) at any t within the five hour period ($0 \le t \le 300$) using the following formula:

$$x(t) = \sum_{n=0}^{10} x(nT)\operatorname{sinc}\left(\frac{t-nT}{T}\right)$$

   a. Now, you had the following 11 readings in your notebook:

x(260)	x(264)	x(268)	x(272)	x(276)	x(280)	x(284),	x(288)	x(292)	x(296)	x(300)
-0.266	-0.168	-0.058	0.060	0.180	0.297	0.403	0.491	0.553	0.586	0.588

   Putting these values on the left hand side of the above equation, you formulate a system of 11 equations with 11 unknowns[1] x(0), x(T), x(2T), …, x(10T). Use Matlab to find these eleven unknowns.

   b. After giving yourself a few "high-fives", you suddenly realize that something must be wrong – it means that the readings throughout five hours can be reconstructed using the readings captured within the last half an hour. In fact, it is not hard to see that they can be reconstructed using *any eleven readings captured within an arbitrary small period of time*! How can this be?

[1] You actually know x(10T) = x(300) already but this is not important for this question.