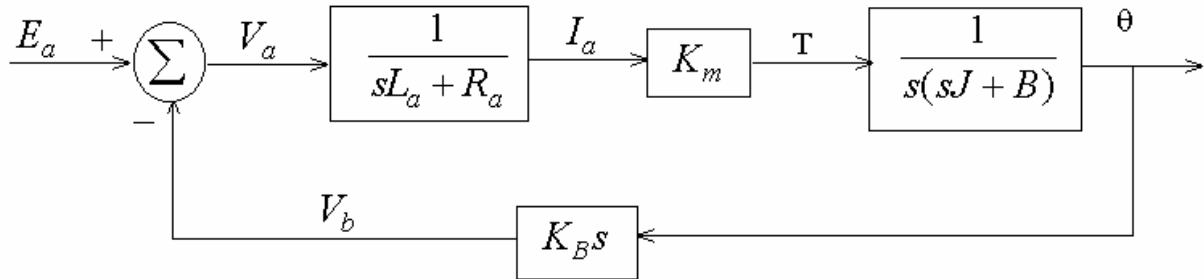
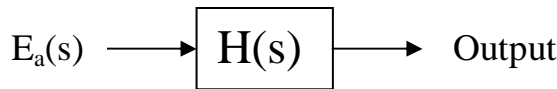


6.6 Block Diagrams and Feedback Systems

What is a block diagram? Composition of modular subsystems.



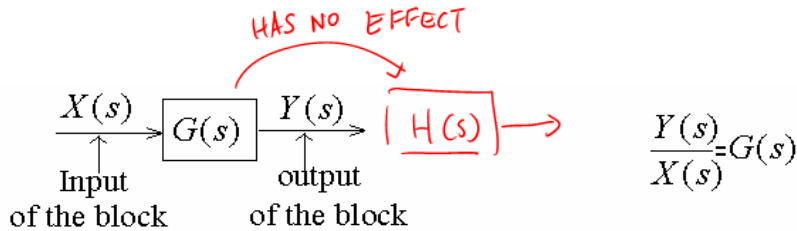
Want to represent the whole system as



so that we can study its stability, frequency response, time response, etc.

Question: Can we just study them separately?

Basic Block

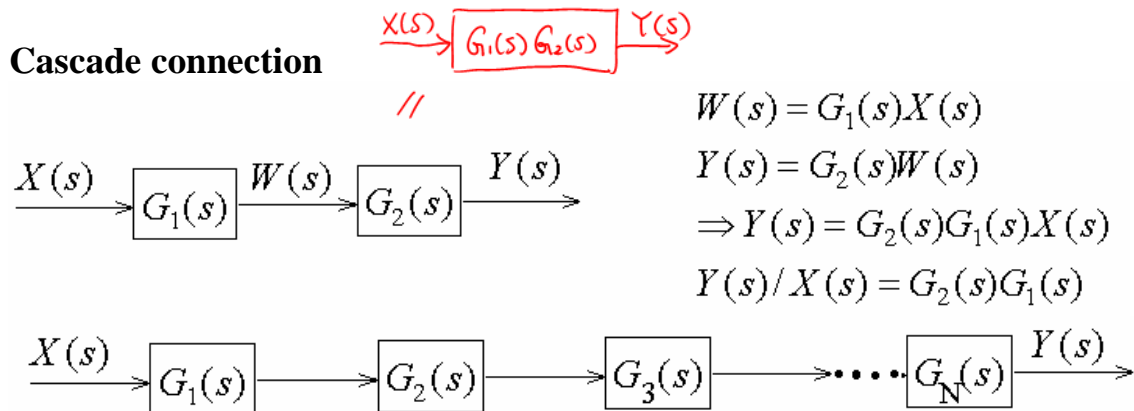


Assumption: Y(s) is determined by input X(s) and block transfer function G(s).

* The output can have arbitrary fan-out and G(s) will have no effect on the input.

* These assumptions might not hold in real applications. We'll see an example later.

Cascade connection



$G_1(s) = \frac{s-1}{\dots}$, $G_2(s) = \frac{\dots}{s-1}$ $Y(s)/X(s) = ?$

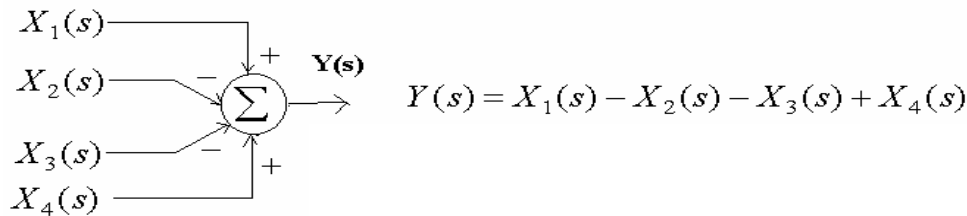
↑
Not always true because of pole-zero cancellation!



Q: If G_1, G_2, \dots, G_N are all asyn. stable, is $G_1 G_2 \dots G_N$ stable also? **YES**

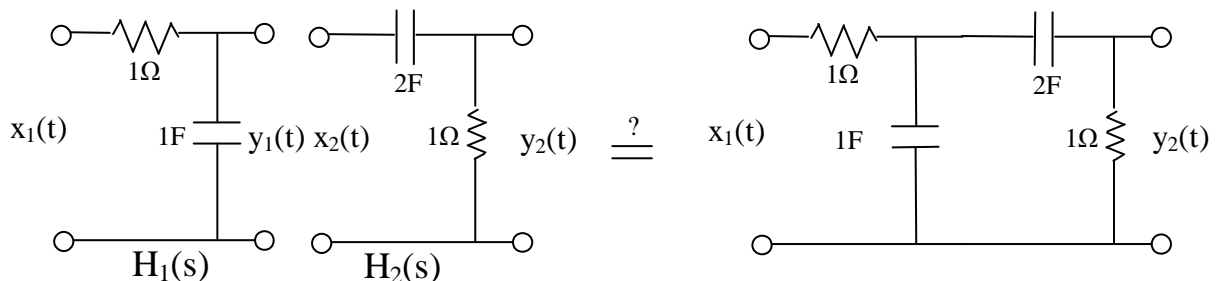
← Q: Is the inverse true? i.e. if $G_1 G_2 \dots G_N$ is stable, are all G_1, G_2, \dots, G_N stable?

Summer



Example: Loading Problem

Consider the two RC networks shown in the left diagram below.



Using impedances, we can easily compute: $H_1(s) = \frac{1}{s+1}$ and $H_2(s) = \frac{2s}{2s+1}$

Cascade rule implies that the overall system is thus

$$H(s) = H_1(s)H_2(s) = \frac{1}{s+1} \cdot \frac{2s}{2s+1} = \frac{2s}{2s^2 + 3s + 1} \tag{1}$$

On the other hand, the right diagram of the actual cascade circuit. Again it is easy for us to compute the transfer function (please work it out yourself):

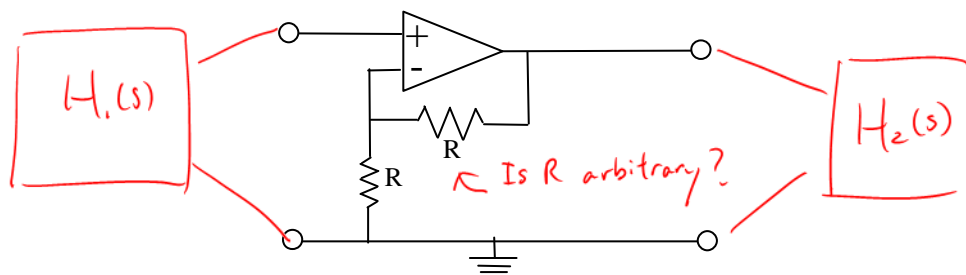
$$H(s) = \frac{Y_2(s)}{X_1(s)} = \frac{2s}{2s^2 + 5s + 1} \tag{2}$$

Question 1: (1) and (2) are clearly different. Why?

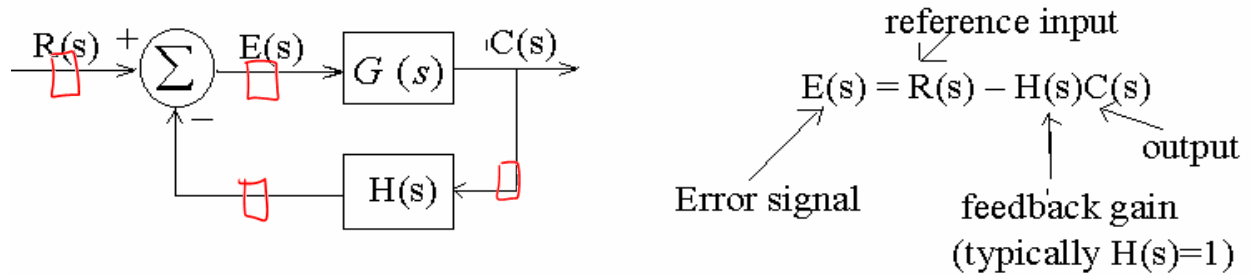
Answer: The ideal block model assumes that the input port of the second block draws no current from the output port of the first block. It's clearly not the case here. This is called the *Loading Problem*.

Question 2: Can you make the circuit behave more like the ideal case?

Answer: Put a *buffer* or *isolating amplifier* between blocks.



Feedback system:



$G(s)$: system to be controlled

Let's find $T(s) = C(s)/R(s)$ Closed-loop transfer function

Equation (1) $C(s) = G(s)E(s)$ ✓

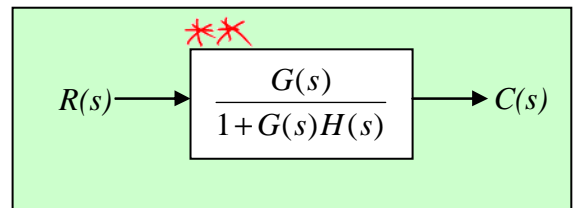
Equation (2) $E(s) = R(s) - H(s)C(s)$

$$C(s) = G(s)(R(s) - H(s)C(s))$$

$$= G(s)R(s) - G(s)H(s)C(s) \checkmark$$

$$\Rightarrow (1 + G(s)H(s))C(s) = G(s)R(s)$$

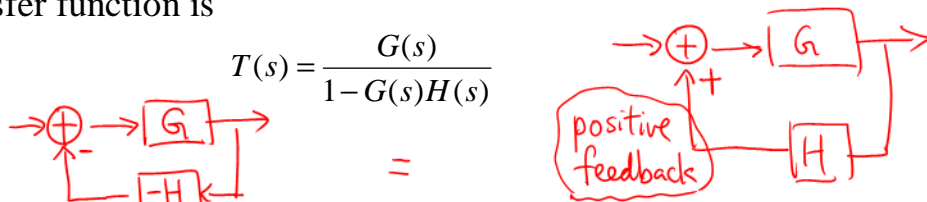
$$\Rightarrow T(s) = C(s)/R(s) = \frac{G(s)}{1 + G(s)H(s)}$$



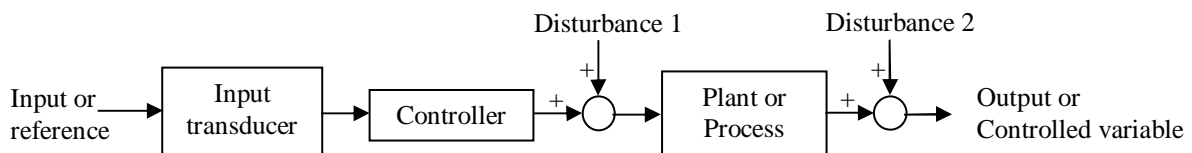
This is a negative feedback system. For positive feedback, we replace $H(s)$ by $-H(s)$ and the transfer function is

$$T(s) = \frac{G(s)}{1 - G(s)H(s)}$$

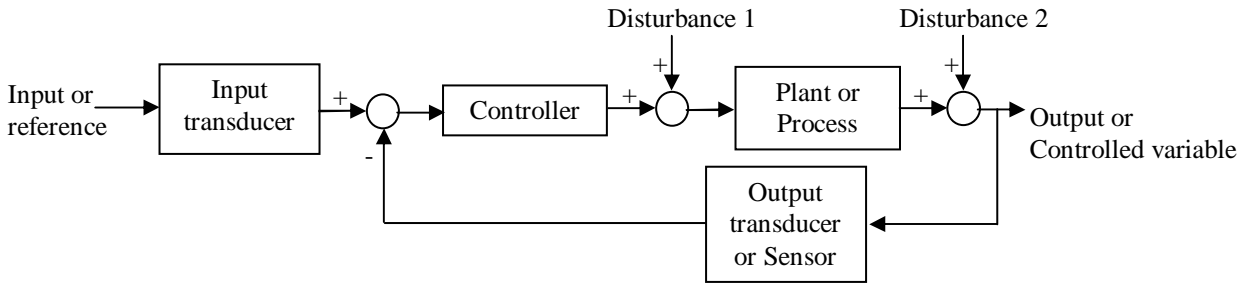
Why feedback?



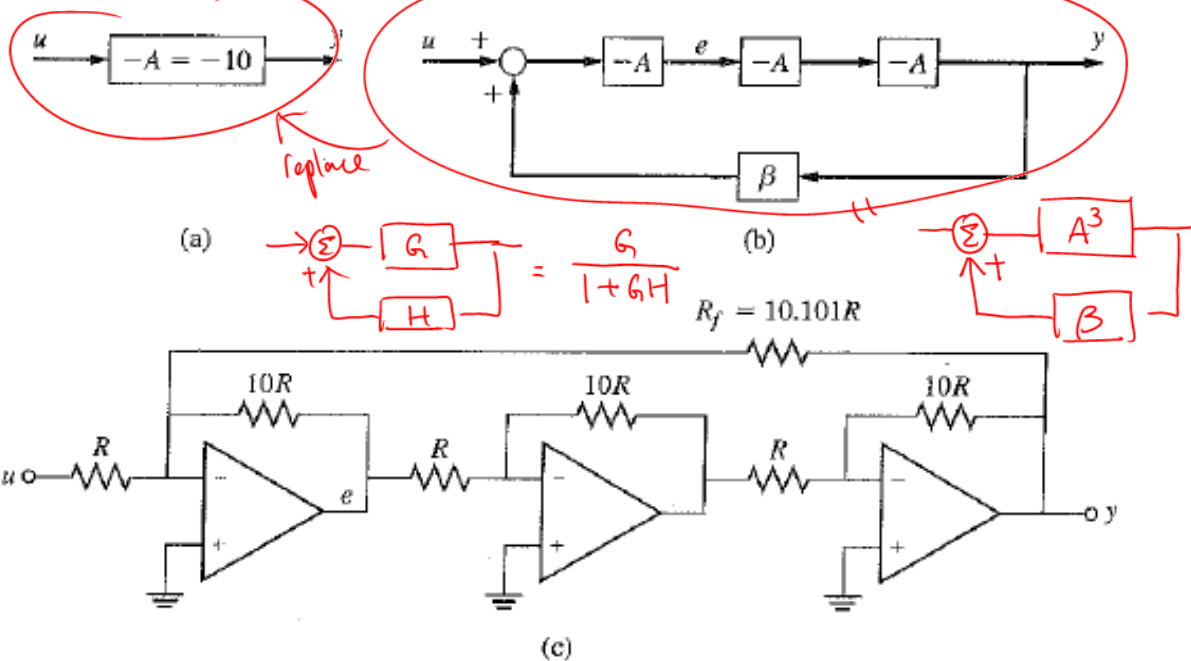
Closed-loop system is extremely useful in control because it allows a feedback path for adjustment. An open-loop system like the one below cannot compensate for any disturbances accumulated at the controller and the output:



On the other hand, by using a feedback loop that captures the output, the system can adjust the input or reference to compensate for any disturbance.



Example: Robust Amplifier



You need an amplifier of gain $A=10$ as shown in Figure (a) – the negative sign follows the inverting gain convention used in the book that I got this figure from.

Let's say you have only access to poor-quality components so that the gain A reduces 10% every year. To combat such decay, you try a system with three amplifiers plus a positive feedback shown in Figure (b). Using the feedback formula, we have

$$A_f \equiv H(s) = \frac{A^3}{1 + \beta A^3} = \frac{1}{1/A^3 + \beta} = 10 \Rightarrow \beta = 0.099$$

$A=0$

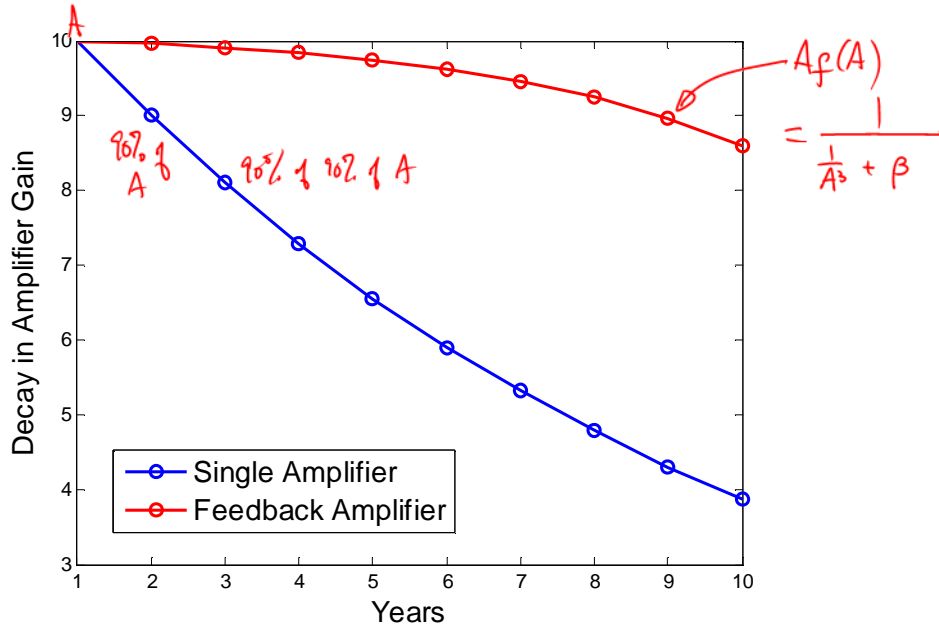
In order to have $H(s) = A = 10$, we need to set β to 0.099:

$$\frac{10^3}{1 + \beta 10^3} = 10 \Rightarrow \beta = 0.099$$

Note that the gain β does not require an extra amplifier – it can be implemented by adjust the relative resistance of the inverting amplifier. (Exercise).

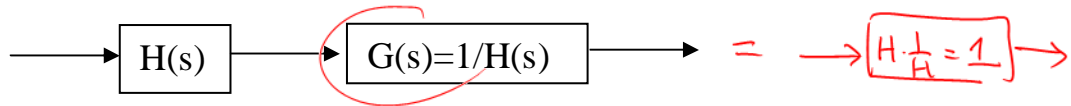
Even though the positive feedback uses three times as much component, the overall gain $A_f = \frac{1}{1/A^3 + \beta}$ makes it very insensitive to changes in A.

We can see this by plotting the gains for the two systems over ten years:



Example: Approximation of Inverse

Given a system H(s), if we want to undo its effect, we can put form a cascade system like the following:

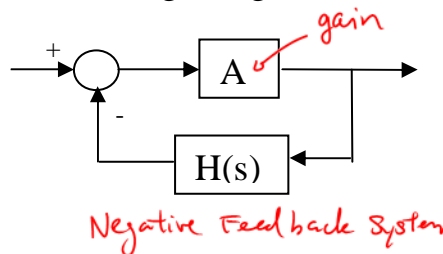


This may not always work because

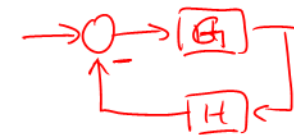
1. H(s) may have zeros on the open right half plane.
2. The inverse system relies on VERY PRECISE Cancellation of poles and zeros between H(s) and G(s) – not practical in real-life.

Instead, we can approximate it using a negative feedback system:

Try to build an inverse of H(s)



$$G(s) = \frac{A}{1 + A H(s)}$$



$$T(s) = \frac{G}{1 + HG}$$

The overall system is thus

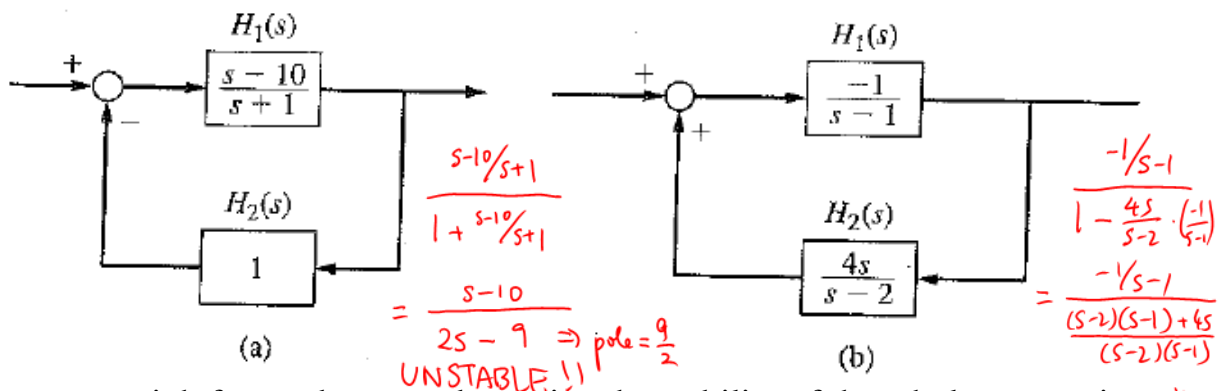
$$H_o(s) = \frac{A}{1 + AH(s)} \approx \frac{1}{H(s)} \quad \text{if } |AH(s)| \gg 1$$

accomplished by using a very large gain of A

Of course, we also need to check that $H_o(s)$ is stable.

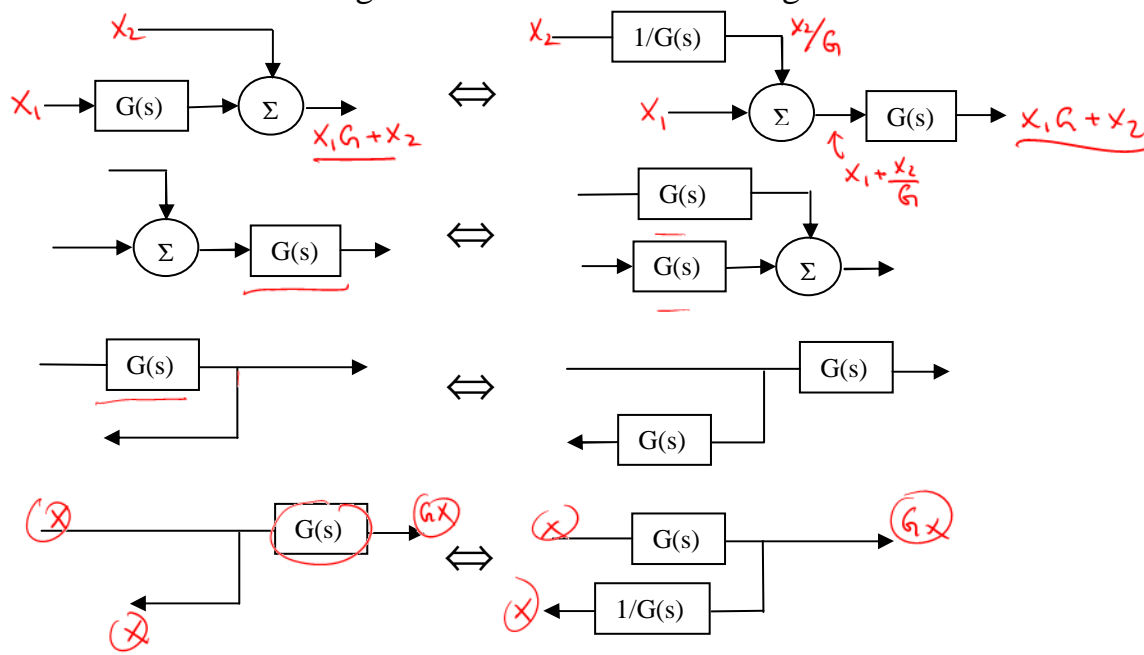
(Asymptotic) Stability of Composite System

Consider the following two examples: the left system is unstable even though all the subsystems are stable. The right system, on the other hand, is stable even though all subsystems are unstable.

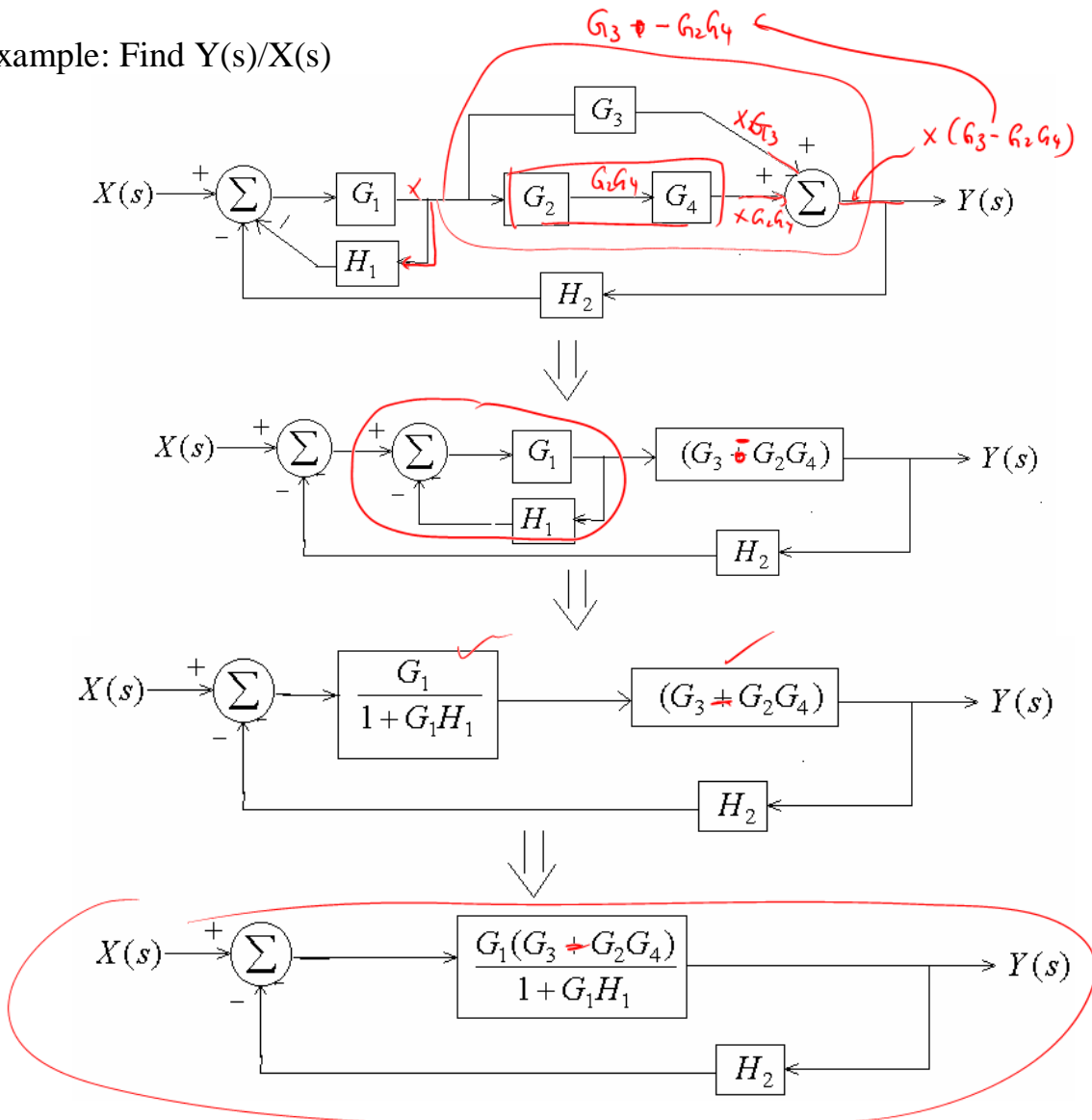


The most straightforward way to determine the stability of the whole system is to combine them into a single transfer function. There are techniques (Nyquist Stability Criterion) that can determine the stability of negative feedback systems without computing the transfer function but we will not cover them in this class.

Idea: combine blocks together to form familiar configurations.



Example: Find $Y(s)/X(s)$



$$\frac{Y(s)}{X(s)} = \frac{\frac{G_1(G_3 + G_2G_4)}{1 + G_1H_1}}{1 + \frac{G_1(G_3 + G_2G_4)}{1 + G_1H_1} H_2} = \frac{G_1(G_3 + G_2G_4)}{1 + G_1H_1 + H_2G_1(G_3 + G_2G_4)}$$