Name: ______________________________________

Midterm 2 (Spring 2007) of EE422G:

1. This midterm consists of six single-sided pages. The first page has the Z-transform tables and the last is an extra worksheet. You can tear out any page but make sure all the pages you turn in have your name and student ID on them.
2. There are four problems in this exam. You have 1.25 hours to finish this exam.
3. You are allowed to use one double-sided page of cheat sheet.

Good luck!

	Properties of Z-transform		
1	Linearity	$a_1x_1(nT)+a_2x_2(nT)$	$a_1X_1(z)+a_2X_2(z)$
2	Multiply by a^n	$a^n x(nT)$	$X\left(\frac{z}{a}\right)$
3	Time Delay	$x(nT-mT)u(nT-mT), m>0$	$z^{-m}X(z)$
4	Multiply by n	$nx(nT)$	$-z\frac{d}{dz}X(z)$
5	Initial Value Theorem	$x(0)=\lim_{z\to\infty}X(z)$	
6	Final Value Theorem	$x(\infty)=\lim_{z\to 1}(1-z^{-1})X(z)$	
7	Time Convolution	$\sum_{m=0}^{\infty}x(mT)y(nT-mT)$	$X(z)Y(z)$

TABLE 8-1
Short Table of z-Transforms

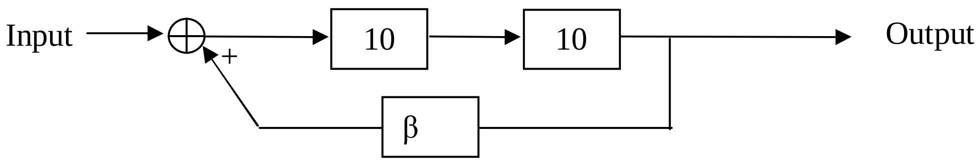
Transform Pair Number	Continuous-time Function $f(t)$ for $t>0$	Sample Values $f(nT)$ for $n \geq 0$	z-Transform of $f(nT)$
1.	—	$f(nT)=\begin{cases}1, & n=0\\ 0, & n\neq 0\end{cases} \triangleq \delta(n)$	1
2.	1 (unit step)	1	$\frac{1}{1-z^{-1}}$
3.	e^{-at}	$e^{-anT}=(e^{-aT})^n=K^n$	$\frac{1}{1-e^{-aT}z^{-1}}=\frac{1}{1-Kz^{-1}}$
4.	t	nT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
5.†	te^{-at}	nTe^{-anT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
6.†	$\sin bt$	$\sin bnT$	$\frac{(\sin bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
7.†	$\cos bt$	$\cos bnT$	$\frac{1-(\cos bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
8.†	$e^{-at}\sin bt$	$e^{-anT}\sin bnT$	$\frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$
9.†	$e^{-at}\cos bt$	$e^{-anT}\cos bnT$	$\frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$

†In transforms 5 through 9, e^{-aT} and bT can be replaced by constants, K_1 and K_2, respectively, as was done in transform 3. Convergence of the z-transform requires $|K_1| < 1$.

Name: ____

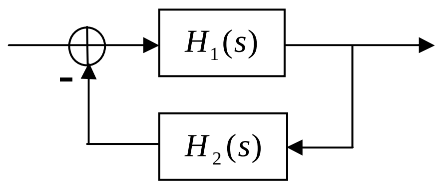
1. (25 points) Feedback System & Routh Array

a) (7 points) Compute the appropriate β so that the following robust amplifier will give a gain of 10.



The transfer function is $H(s) = \dfrac{100}{1-100\beta}$. To set H(s)=10, we must have β=-0.09.

b) (7 points) You are given an unstable system $H_1(s) = \dfrac{10}{s-1}$. Suggest a simple $H_2(s)$ to be used as a negative feedback so that the overall system $H(s)$ is i) BIBO stable and ii) the magnitudes at DC are the same, i.e. $|H(0)| = |H_1(0)|$



A particularly simple one is to use a simple gain for $H_2(s)=\beta$. The transfer function of the feedback system is

$$H(s) = \frac{H_1(s)}{1+H_1(s)H_2(s)} = \frac{10}{s-1+10\beta}$$

Any β>0.1 can do the job. Using β = 0.2 will provide the desired DC gain.

c) (4 points) Answer the following two questions about part a) and part b).

(i) How to further extend the useful lifespan of the amplifier in part b)

(ii) Name another application of a NEGATIVE FEEDBACK system.

i) Increase the number of amplifier with gain 10; ii) Approximate inverse system.

d) (7 points) Fill in the missing TWO rows of the Routh Array that corresponds to the following transfer function

$$H(s) = \frac{1}{s^7 + 3s^6 + 3s^5 + s^4 + s^3 + 3s^2 + 3s + 1}$$

s^7	1	3	1	3
s^6	3	1	3	1
s^5	8/3	0	8/3	
s^4	1	0	1	
s^3	4	0		
s^2	ε	1		
s^1	-4/ε	0		
s^0	1			

Name: ______________________________________

2. (25 points) Sampling and Interpolation

a. (4 points) State the Nyquist Theorem

Nyquist Theorem states that a continuous-time band-limited signal can be perfectly reconstructed from its discrete samples if the sampling frequency is twice the bandwidth.

b. (7 points) Explain why linear interpolation is a better D/A technique than sample-and-hold.

Linear interpolation is better than sample-and-hold as the frequency response of linear interpolation decays faster than sample-and-hold and thus better suppress the high frequency components introduced by sampling.

c. (7 points) A continuous-time signal $x_c(t)=\cos(4000\pi t)$ is sampled with sampling period T to obtain the discrete-time signal $x_c(nT) = x_d(n) = \cos(\pi n/3)$. Determine a choice of T that is consistent with this information.

Since $x_d(n) = x_c(nT)$, we have $4000\pi nT = \pi n/3 \Rightarrow T = \frac{1}{12000}$

d. (7 points) Continued from part c., is your choice for T unique? If so, explain why. If not, specify another choice of T.

T is not unique because $\cos(4000\pi nT) = \cos(\frac{\pi n}{3} + 2m\pi)$ for any integer m. In particular we can choose m = n, and have $4000\pi nT = \pi n/3 + 2n\pi \Rightarrow T = \frac{7}{12000}$

Name: ____

3. (25 points) Z-transform and Discrete-Time Linear System

a. (5 points) Find the Z-transform of $x(nT) = \left(\frac{1}{2}\right)^{n-1} u(nT - T)$ and specify its region of convergence.

Since $Z\left[\left(\frac{1}{2}\right)^n u(nT)\right] = \frac{1}{1-0.5z^{-1}}$, then $X(z) = \frac{z^{-1}}{1-0.5z^{-1}}$. Its ROC is {z:|z|>0.5}.

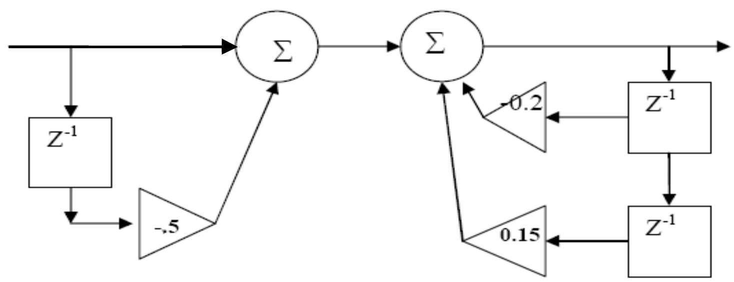
b. (5 points) Find the inverse Z-transform of $X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}$

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-2}} = \frac{1}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}}$$

Solving the partial fraction, we got A=0.5, B=0.5 and hence

$$x(nT) = \left[\left(\frac{1}{2}\right)^{n+1} - \left(\frac{-1}{2}\right)^{n+1}\right] u(nT)$$

c. (10 points) Find the discrete-time system H(z) implemented based on the following block diagram

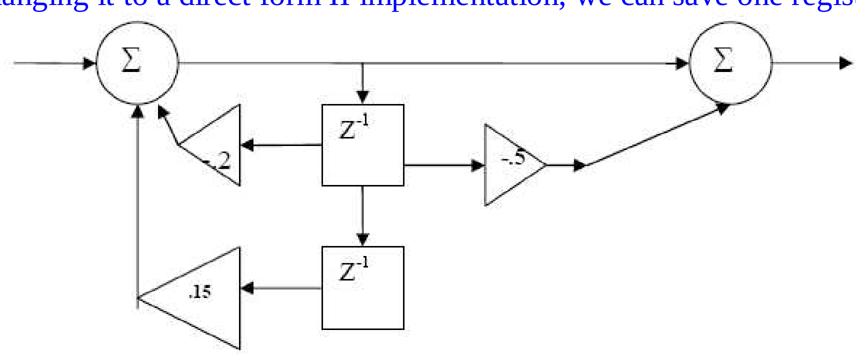


This is clearly a direct form I implementation. Thus the left part describes the feed-forward portion (numerator) and the right part describes the feedback protion (denominator). Thus,

$$H(z) = \frac{1-0.5z^{-1}}{1+0.2z^{-1}-0.15z^{-2}}$$

d. (5 points) Can you draw an alternative design (block diagram) that uses fewer than three delay registers?

By changing it to a direct form II implementation, we can save one register.



Name: ____

4. (25 points) Discrete-Time Fourier Transform

a) (5 points) Given the DTFT of a LTI system is $H(e^{j\omega T}) = \sum_{n=0}^{\infty} h(nT)e^{-j\omega Tn}$, show that if the impulse response h(nT) is real, we have $H(e^{-j\omega T}) = \overline{H(e^{j\omega T})}$, i.e. the negative frequency content can be deduced by taking the conjugate of the positive frequency content.

We can go straight from the definition:

$$\begin{aligned} H(e^{-j\omega T}) &= \sum_{n=0}^{\infty} h(nT)e^{j\omega Tn} \\ &= \overline{\sum_{n=0}^{\infty} h(nT)e^{j\omega Tn}} = \overline{H(e^{j\omega T})} \end{aligned}$$

b) (5 points) If the input to $H(e^{j\omega T})$ is $x(nT) = \cos(\omega_0 nT) = \frac{1}{2}(e^{j\omega_0 nT} + e^{-j\omega_0 nT})$, use part a) to show that the output is given by

$$y(nT) = |H(e^{j\omega_0 T})| \cos\left(\omega_0 nT + \angle H(e^{j\omega_0 T})\right)$$

As $x(nT) = \cos(\omega_0 nT) = \frac{1}{2}(e^{j\omega_0 nT} + e^{-j\omega_0 nT})$ and we know that the complex exponential is the "eigen-signal" of any LTI system. Thus, the output

$$\begin{aligned} y(nT) &= \tfrac{1}{2}\left(H(e^{j\omega_0 T})e^{j\omega_0 nT} + \overline{H(e^{j\omega_0 T})}e^{-j\omega_0 nT}\right) \\ &= |H(e^{j\omega_0 T})| \cdot \tfrac{1}{2}\left(\exp\left(j\left(\omega_0 nT + \angle H(e^{j\omega_0 T})\right)\right) + \exp\left(j\left(-\omega_0 nT - \angle H(e^{j\omega_0 T})\right)\right)\right) \\ &= |H(e^{j\omega_0 T})| \cos(\omega_0 nT + \angle H(e^{j\omega_0 T})) \end{aligned}$$

c) (7 points) A digital system is defined by the difference equation

$$y(nT) = \frac{1}{2} y(nT - T) + 3x(nT)$$

Find $H(e^{j\omega T})$.

$$H(z) = \frac{3}{1 - \frac{1}{2} z^{-1}} \Rightarrow H(e^{j\omega T}) = \frac{3}{1 - \frac{1}{2} e^{-j\omega T}}$$

d) (8 points) If the input to the system in c) is $x(nT) = \cos(\pi n)$, use part b) to find the output y(nT).

First we have $\omega_0 T = \pi$, thus $H(e^{j\omega_0 T}) = \dfrac{3}{1 - \frac{1}{2} e^{-j\pi}} = \dfrac{3}{1 + \frac{1}{2}} = 2$

Thus $y(nT) = 2\cos(\pi n)$

Name: ____

EXTRA WORKSHEET