Welcome to EE440, please pick up handout up front.

**Single, Scalar Random Variables**

Discrete R.V. ex Coin toss

\[ P(\text{Head}) = 0.3 \]

\[ P(\text{Tail}) = 0.7 \]

Continuous R.V. ex Gaussian, Exponential

But in the real world, we have A LOT MORE random variables.

DNA - a sequence of DNA bases: A, G, C, T

ex: DNA - \( x_1 \), \( x_2 \), \( x_3 \), ..., \( x_n \)

\( x_i = \text{A or G or C or T} \)

\[ P(x_1 = \text{A}, x_2 = \text{G}, x_3 = \text{A}, \ldots, x_n = \text{T}) = 0.0001 \text{ requires 1 byte} \]

How big is this table \( P \)? \( 4^n - 1 \) bytes

1 GB memory ⇒ what is the biggest \( n \) that can be stored?

\[ n = 15 \]

A typical human chromosome: \( n = 220 \text{ million} \)

The key goal of this course is to study computationally efficient techniques to represent correlation structures between many random variables (≈ stochastic processes)

The other extreme: independent.

\[ P(x_1, x_2, x_3, ..., x_n) = P(x_1) P(x_2) P(x_3) ... P(x_n) \]

\[ \text{requires } 4^n - 1 \text{ bytes} \]

\[ P(A) = 0.1 \]

\[ P(C) = 0.5 \]

\[ P(T) = 1 - 0.1 - 0.3 - 0.5 = 0.1 \]

\[ \text{3 Don't need to store} \]

1 GB ⇒ \( n = 268 \text{ million} \)
Mathematical Proof

What is a proof?

To show a statement to be true based on

DEDUCTIVE REASONING from axioms or definition

OR

To disprove a statement by providing ONE COUNTEREXAMPLE.

Axioms / Definitions \( \rightarrow \) statements

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<tr>
<th>P</th>
<th>Q</th>
<th>( P &amp; Q )</th>
<th>( P \lor Q )</th>
<th>( P \Rightarrow Q )</th>
<th>( \neg P )</th>
<th>( P \Leftrightarrow Q )</th>
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A typical proof: Axioms \( \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \ldots \Rightarrow S_n \Rightarrow P \)

1. Direct Proof of a statement \( P \)

\[
[(Q \Rightarrow P) \& Q] \Rightarrow P
\]

\( Q \) implies \( P \)

Statement known to be true
Slight variation: contrapositive

\[(P \Rightarrow Q) \iff (\neg Q \Rightarrow \neg P)\]

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Identical ✓

Example: Prove the principle of Mathematical Induction

Statement: Sum of \(1 + 2 + 3 + \ldots + n = \frac{(1+n)n}{2}\) \(S(n)\) \(n \in \text{Natural numbers} \ \{1, 2, 3, \ldots \}\)

Base case: Prove \(S(1)\) to be true.

\[\text{LHS} = 1\]
\[\text{RHS} = \frac{(1+1)(1)}{2} = 1 \quad \checkmark\]

Induction step: \(S(k) \& S(k) \Rightarrow S(k+1)\) for all \(k \in \mathbb{N}\) \((\forall k \in \mathbb{N})\)

\(S(k)\) is true, \(1 + 2 + 3 + \ldots + k = \frac{(1+k)k}{2}\)

Consider \(S(k+1)\), \(LHS = 1 + 2 + 3 + \ldots + k + k+1\)

\[\text{LHS} = 1 + 2 + \ldots + k + 1\]
\[\text{RHS} = \frac{(1+(k+1))(k+1)}{2} = \frac{(k+2)(k+1)}{2} = \frac{(k+1)(k+2)}{2} = \text{RHS} \quad \text{by assumption}\]
0x2/ Why Mathematical Induction work?

M.I : \( P \Rightarrow Q \)

\( P = \text{"} S(1) \text{ is true and } S(k) \Rightarrow S(k+1) \text{ } \forall k \in \mathbb{N} \text{"} \)

\( Q = \text{"} S(n) \text{ is true } \forall n \in \mathbb{N} \text{"} \)

Prove by contrapositive \( (\neg Q \Rightarrow \neg P) \)

\( \neg Q = \text{"} S(n) \text{ is false for at least one } n \in \mathbb{N} \text{"} \)

\( \text{"} \exists n \in \mathbb{N} \text{ such that } S(n) \text{ is false"} \)

\( \neg P = \text{"} S(1) \text{ is false OR } (\exists k \in \mathbb{N} \text{ } S(k) \& \neg S(k+1)) \text{"} \)

Continue next time