# SYMMETRIC SHAPE COMPLETION UNDER SEVERE OCCLUSIONS

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**ABSTRACT**

In this paper, we propose a novel algorithm for completing rotationally symmetrical shapes under severe occlusions. The intuitive idea is to use the existing contour, under a carefully estimated similarity transform, to fill in the missing portion of a symmetric object due to occlusions. Our algorithm exploits the invariant nature of the curvature under similarity transform and the periodicity of the curvature of a symmetric object contour. To arrive at the appropriate transform, we first estimate the fundamental period in the curvature. We use the fundamental period and the harmonic components to estimate the fundamental angle of rotation and the centroid of the unoccluded shape, which in turn establish different modes of symmetry. By following each mode of symmetry we compute the corresponding transform and select the ones that best complete the missing portion of the contour.

***Index Terms—*** Image shape analysis, Interpolation

## 1. INTRODUCTION

Symmetry is one of the most important pervasive cues that can be observed in most of natural as well as man-made environments. The concept of symmetry has therefore attracted considerable attention and much research efforts have been devoted to analyze and quantify the properties of symmetric structures [1]. Contour completion and reconstructing symmetric objects under severe occlusions offer tremendous opportunities in many areas of computer vision applications such as digital inpainting, machine vision of robots, object recognition and identification.

One of the important applications of contour completion is in the area of image inpainting. Image inpainting is a technique to fill the missing region, or the *hole*, based on the surrounding image statistics [2, 3]. A majority of the inpainting techniques attempt to inpaint by propagating local surrounding information into the hole region. However they do not take into account the global attributes available throughout the image which might offer some important structural cues. We believe that structural completion plays a vital role in providing a perceptually complete inpainting and a global inpainting algorithm incorporating such measure will be an effective one. Most of the contemporary contour completion schemes employ energy minimization functional or Partial Differential Equation (PDE) based approach without explicitly taking advantage of any object symmetry [4, 5]. Using object symmetry to complete occluded or missing object contour is a relatively unexplored area in computer vision. In [6], Zabrodsky et al. describe various symmetry structures and define a continuous symmetry measure referred to as "symmetry distance" for evaluating different types of symmetry. They use this distance measure to reconstruct the symmetric shape similar to the original occluded contour. Nonetheless, their approach requires an a-priori determined order of rotational symmetry for completing the missing structure.

In this paper, we propose a novel algorithm for rotationally symmetric shape completion in the presence of severe occlusions. Unlike [6], no a-priori knowledge is needed. We utilize the invariant nature of the curvature against rotations and translations of symmetric objects to complete the missing regions of the contour. The rest of the paper is organized as follows: in Section 2, we explain the process of estimating the fundamental angle of rotation and centroid by utilizing the periodic nature of the curvature and present experimental results. In section 3, we discuss the use of this algorithm in a global inpainting application. Finally we conclude the paper in Section 4.

## 2. METHODOLOGY

Consider the partially occluded equilateral hexagon shown in Figure 1(a). An intuitive way to perform completion is to rotate and translate the original contour around the centroid of the unoccluded shape so as to match the missing portion and form a symmetric hexagon. It is a non-trivial problem because, under severe occlusion, the centroid of the occluded object can be far away from that of the unoccluded object [6]. In the following two sections, we describe our approach of using the curvature of the contour to estimate both the fundamental angle of ration and the centroid of the unoccluded shape.

### 2.1. Estimation of Fundamental Angle of Rotation

We treat the input curve as an open contour which is represented as a sequence of $n$ points $(x_1, y_1)$, $(x_2, y_2)$, $\ldots$, $(x_n, y_n)$ following a particular orientation. The $x$ and $y$ coordinates of the pixels are parameterized by the curve arc-length parameter $u$, and $u$ is normalized to take values from the interval $[0, 1]$. The functions $x(u)$ and $y(u)$ are then resampled to $N$ equidistant points using a cubic spline interpolation. We use $N = 256$ which is found to be reasonable for typical image processing applications. The resampled function $x(t)$ and $y(t)$ is low-pass filtered using a normative Gaussian filter to obtain a smoothed contour. We then compute the curvature of the contour as follows:

$$\kappa(t) = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \quad (1)$$

where the dots indicate differentiation with respect to $t$ and the discrete parametrization of the contour is $\{(x(t), y(t))\}$ where $t = 0, 1, \ldots, N-1$. The computed curvature curve is shown in Figure 1(b). We also compute the normal vector $\mathbf{n}(t)$ at each point on

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the curve as follows:

$$\mathbf{n}(t)=\frac{\mathbf{e}(t)}{||\mathbf{e}(t)||} \text{ where } \mathbf{e}(t)=(\ddot{x},\ddot{y})-\frac{(\ddot{x}\dot{x}+\ddot{y}\dot{y})}{(\dot{x}^2+\dot{y}^2)}(\dot{x},\dot{y}) \quad (2)$$

The normal vectors will later be used to compute the *fundamental angle of rotation* – the smallest angle of rotation of the unoccluded object about its centroid so that it returns to its original position.

Using the arc-length parametrization, it can be easily shown that *a rotation about the centroid of the unoccluded object* manifests as a translation of the curvature curve [7]. Since the contour realigns itself after rotating an integral number of the fundamental angle, the curvature curve of a rotationally symmetric contour must be periodic. We further assume that the visible contour contain at least two periods otherwise the period cannot be estimated. To robustly estimate this period $T$ of the curvature curve, we employ a sliding-window based technique. First, we select a $N/2$-point *search segment* from the curvature curve at a random starting point $t=q$ to $t=N/2+q-1$. Second, among all the $N/2$-point segments from the curvature curve, we identify the segment $\hat{\tau}$ points away from the search segment that maximizes the autocorrelation:

$$\hat{\tau}=\max_{\tau\in S}\sum_{t=q}^{N/2+q-1}\kappa(t)\kappa(t+\tau) \quad (3)$$

where $S=\{-q,\ldots,q-1,q+1,\ldots,N/2-q\}$. $\hat{\tau}$ must be in the form of $kT$ where $k$ is a positive integer. As neighboring structures tend to be more correlated than their distant counterparts, $k$ is typically 1. To ensure a robust estimate, we randomly select multiple search segments and estimate $T$ based on the smallest computed $\hat{\tau}$. We then identify all pairs of curvature points that are an integral number of $T$ from each other. Let the number of correspondence be $M$. Each correspondence $(x(t_i),y(t_i))\leftrightarrow(x(t_i+k_iT),y(t_i+k_iT))$ for $i=0,1,\ldots,M-1$ is parameterized by the index $t_i$ of the first point and the number of period $k_i$ the second point from the first.

Once the correspondences are established, we can estimate the fundamental angle of rotation $\theta$ by computing the angle between the normal vectors of the corresponding points in the original contour. We compute the angle between the normal vectors of all the corresponding points and take the average value as the estimate:

$$\theta=\frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{k_i}\cos^{-1}\left(\frac{\mathbf{n}(t_i)\cdot\mathbf{n}(t_i+k_iT)}{|\mathbf{n}(t_i)||\mathbf{n}(t_i+k_iT)|}\right) \quad (4)$$

The above process is explained in Figure 1(d) where we show normal vectors of the two corresponding points $\mathbf{n}(a)$ and $\mathbf{n}(b)$ separated by the fundamental period. Due to the constraint of the rotational symmetry, $\theta$ (in degrees) must be of the form $\theta=\frac{360}{n}$, where $n$ is an integer. We use this constraint to further refine our estimation.

### 2.2. Centroid Estimation and Cost function

In Section 2.1, we obtain a set of correspondences $(x(t_i),y(t_i))\leftrightarrow(x(t_i+k_iT),y(t_i+k_iT))$ for $i=0,1,\ldots,M-1$. For each correspondence, there exists a rotation transformation matrix $M_{k_i}$ such that

$$M_{k_i}\begin{pmatrix}x(t_i)\\y(t_i)\\1\end{pmatrix}=\begin{pmatrix}x(t_i+k_iT)\\y(t_i+k_iT)\end{pmatrix} \quad (5)$$

$M_{k_i}$ is given by

$$M_{k_i}=\begin{pmatrix}\cos(k_i\theta) & \sin(k_i\theta) & T_x^{k_i}\\-\sin(k_i\theta) & \cos(k_i\theta) & T_y^{k_i}\end{pmatrix} \quad (6)$$

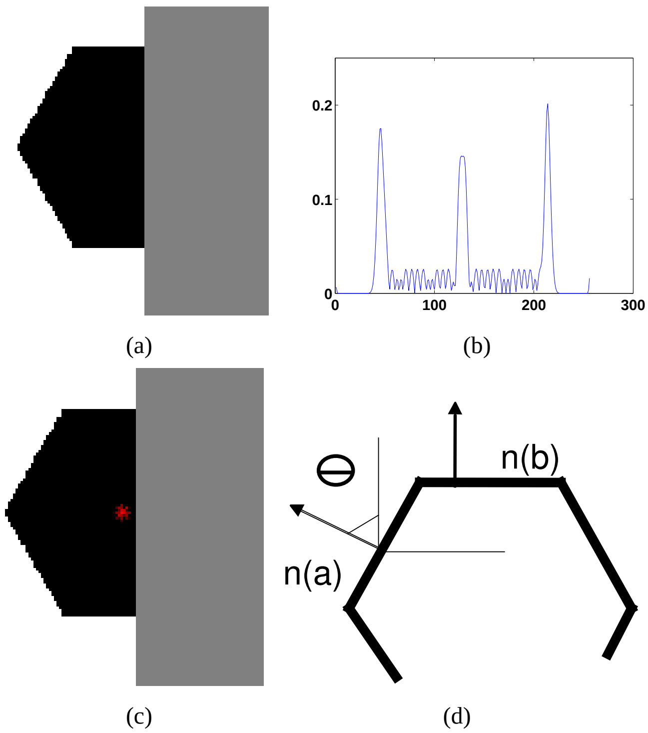


**Fig. 1**. (a) Symmetric hexagon with occlusion; (b) Curvature of the contour of the occluded hexagon; (c) Hexagon with the estimated centroid; (d) Normal vectors of corresponding points $n_x(a)$ and $n_x(b)$ on the contour.

where $T_x^{k_i}$ and $T_y^{k_i}$ are translations in $x$ and $y$ directions.

Since the centroid of the unoccluded shape is the center of rotation, it is a fixed point of $M_{k_i}$ for any integer $k_i$. If the coordinates of the centroid is $(C_x,C_y)$, we must have

$$M_{k_i}\begin{pmatrix}C_x\\C_y\\1\end{pmatrix}=\begin{pmatrix}C_x\\C_y\end{pmatrix} \quad (7)$$

Combining equations (6) and (7), we can eliminate the translation parameters and rewrite Equation (5) as follows:

$$\begin{pmatrix}(1-\cos(k_i\theta)) & -\sin(k_i\theta)\\(1+\sin(k_i\theta)) & -\cos(k_i\theta)\end{pmatrix}\begin{pmatrix}C_x\\C_y\end{pmatrix}=\begin{pmatrix}x(t_i+k_iT)-x(t_i)\cos(k_i\theta)-y(t_i)\sin(k_i\theta)\\y(t_i+k_iT)+x(t_i)\sin(k_i\theta)-y(t_i)\cos(k_i\theta)\end{pmatrix} \quad (8)$$

As we have estimated $\theta$ in the previous section, we can formulate a system of equations based on (8) for all $M$ correspondences and obtain a least square estimation of the location of the centroid.

The final stage of this algorithm involves selecting a suitable candidate from the set of rotations about the centroid to extrapolate the missing contour. Let $\{(\tilde{x}(t),\tilde{y}(t)),t=0,1,\ldots,N-1\}$ be a rotated contour. One end of the rotated contour will align with the original one, while the other end will extrapolate into the missing region and possibly connect back to the opposite end of the original contour. Assume the indices of the extrapolated portion, in reverse order, are $N-1,N-2,\ldots$ and so forth. We use the following cost function to measure how well the extrapolated contour aligns with the unmatched end of the original contour:

$$C(\{\tilde{x},\tilde{y}\}) = \min_{0\leq k\leq N-1} \frac{1}{k+1} \cdot \sum_{t=0}^{k} [\tilde{x}(N-1-t)- \\ x(k-t)]^2 + [\tilde{y}(N-1-t) - y(k-t)]^2 \quad (9)$$

This cost function intuitively measures the distance between the extrapolated region of the transformed contour with the oppoairw end of the original contour segment. The search process is illustrated in the Figure 2. This cost function is computed for all valid candidates, and the suitable candidate is chosen to be the one with the minimal cost. Figure 3 (a)-(c) shows the completion of the occluded hexagon using the first three harmonics. The associated cost for them are 6823.1, 1606.3 and 50.78. Thus, the third harmonics provides the best transformed contour for the completion. If the missing region is too large, it is straightforward to repeat the above process to complete the entire region in a piecemeal fashion.

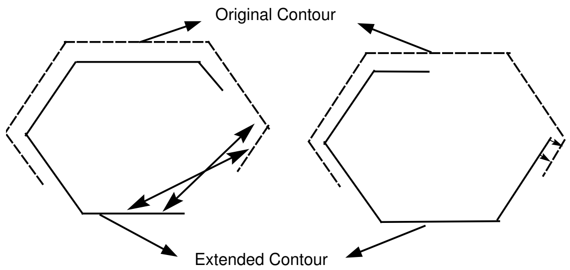


**Fig. 2**. The two figures show the alignment of two candidate co tours with the original one.

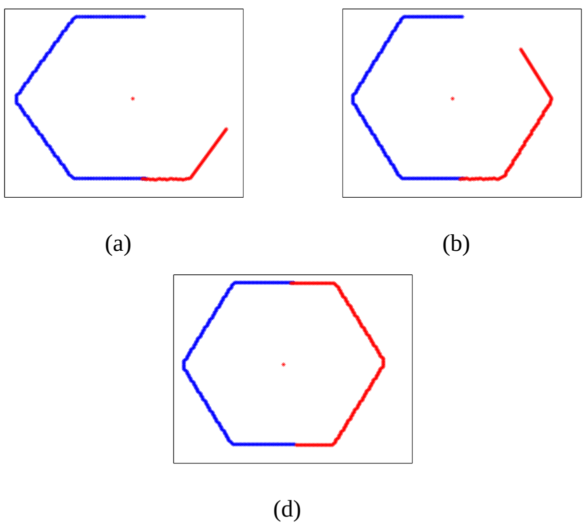


**Fig. 3**. (a) Completion (red) due to rotating the contour by one fundamental period (first harmonic); (b) Completion due to the second harmonic; (c) Completion due to the third harmonic that results in the lowest cost.

In Figure 4, we show another example of our contour completion algorithm. Figure 4(a) shows an occluded shape. We obtain this shape by selecting it from one of the symmetrical clip arts figures from Adobe Photoshop, compute its shape by running an edge detection algorithm and arbitrarily occlude part of the figure. Figure 4(b) shows the curvature curve of the contour and Figure 4(c) indicates the estimated centroid location. By searching for the appropriate transform that minimizes the cost function (9), we find that the second harmonic provides the best match and the result is shown in Figure 4(d). More examples and related software can be downloaded from our website at `http://www.vis.uky.edu/~vijay/research/image.htm`.

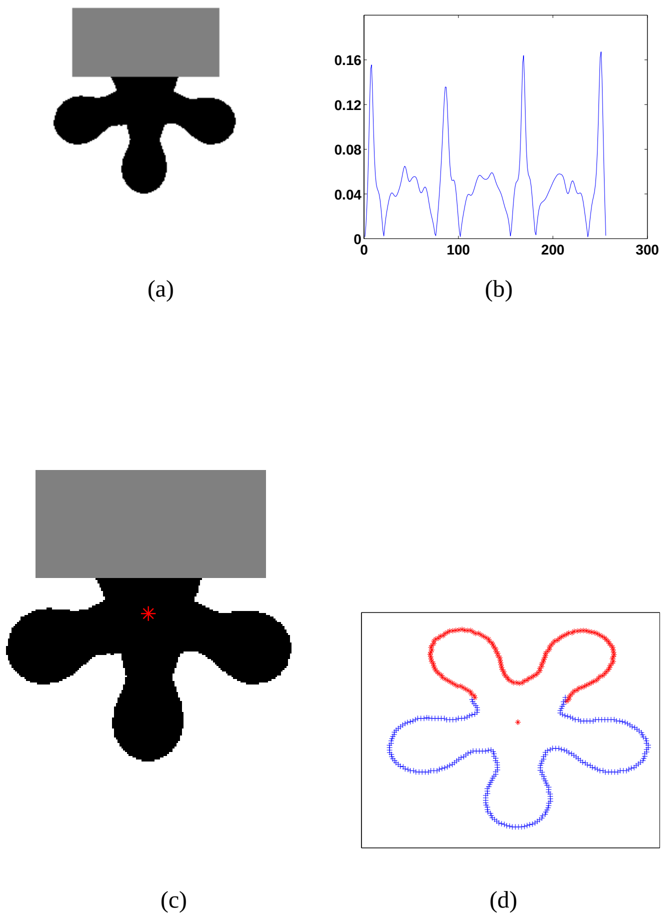


**Fig. 4**. (a) Symmetric object under occlusion; (b) Curvature of the contour of the object; (c) Symmetric object with estimated center; (d) Completed symmetry corresponding to the minimal cost using 2nd harmonic.

## 3. APPLICATIONS TO GLOBAL IMAGE INPAINTING

In this section, we suggest how one can use our curve completion algorithm in performing structural completion task of image inpainting. It has been identified that completing the structure of the underlying image or video object by extension of edges or "isophotes" still remains a challenging task as majority of inpainting algorithms use only local information. Recent efforts in image inpainting have focussed on a two step process, the first stage involving segmentation and structure completion and the second stage by Texture synthesis [8]. We argue that the above symmetry completion algorithm can serve as a useful technique in performing the structure completion. Figure 5(a) shows an image with a hole. Firstly, we segment the image, extract the outer contour and we compute the curvature which is shown in Figure 5(b). We then proceed to estimate the period of the curvature and estimate the centroid shown in Figure 5(c). Finally we select a suitable candidate from a finite set of available candidates by minimizing the cost function defined in Equation 9. The final result of the occlusion completion is shown in Figure 5(d). It is clear that the structure of the occluded region is reconstructed in a perceptually consistent manner. Once underlying structure is completed, we can then utilize effective texture synthesis techniques to fill in the texture details inside the closed boundary to complete the inpainting process.

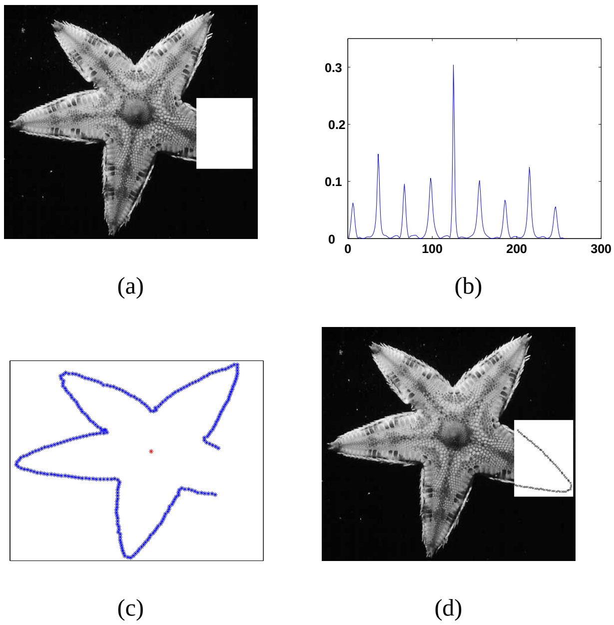


**Fig. 5**. (a) An image with the hole; (b) Curvature of the contour of the segmented object; (c) Contour with estimated center; (d) Structure completion result corresponding to first period and minimal cost

## 4. CONCLUSIONS

In this paper we have presented a rotationally symmetrical shape completion algorithm under the presence of severe occlusions. Our algorithm do not make any assumption about the nature of the circular symmetries to perform the completion and this robust algorithm can be extended to handle various symmetries. The usefulness of this contour completion algorithm is demonstrated in a global inpainting technique by using it for structure completion process. This technique can also be used to complete periodic structures of arbitrary lengths by repeatedly extending the matching segments obtained from the correlation process until it satisfies a minimum cost criterion. In real images, symmetrical objects may not appear to be symmetric due to the projection from three-dimensional world to the two-dimensional image plane. Since the projection can be modeled as a projective transformation, we hypothesize that the above framework can still be used by optimizing the cost function over the space of all projective transforms.

## 5. REFERENCES

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