

## Homework 1 CS 275 Discrete Mathematics Fall 2006

### **Problem 1**

Mr. and Mrs. Richardson want to name their new daughter so that her initials (first, middle, and last) will be in alphabetical order with no repeated initial. How many such triples of initials can occur under these circumstances?

### **Problem 2**

(a) In how many ways can one go from  $(0,0)$  to  $(7,3)$  if the only moves permitted are  $R:(x,y) \rightarrow (x+1,y)$  and  $U:(x,y) \rightarrow (x,y+1)$ , and the number of  $U$ 's may never exceed the number  $R$ 's along the path taken?

(b) Let  $m, n$  be positive integers with  $m > n$ . Answer the question posted in part (a), upon replacing 7 by  $m$  and 3 by  $n$ .

### **Problem 3**

(a) Find the coefficient of  $x^2 y z^2$  in the expansion of  $[x/2 + y - 3z]^5$ .

(b) How many distinct terms are there in the expansion?

(c) What is the sum of all coefficients in the complete expansion?

### **Problem 4**

(a) Determine the number of nonnegative integer solutions to the pair of equations

$$x_1 + x_2 + x_3 = 6 \quad \text{and} \quad x_1 + x_2 + x_3 + x_4 + x_5 = 15$$

(That is, solutions that satisfy both equations simultaneously).

(b) Answer part (a) when both  $=$  signs are replaced by  $\leq$ .

### **Problem 5**

(a) In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is (i) not relevant? (ii) relevant?

(b) Answer part (a) for 18 in place of 17.

### **Problem 6**

As head of quality control, Silvia examined 15 motors, one at a time, and found six defective (D) motors and nine in good (G) working condition. If she listed each finding (of D and G) after examining each individual motor, in how many ways could Silvia's list start with a run of three G's and have six runs in total?

### **Problem 7**

In how many ways can a family of four (mother, father, and two children) be seated at a round table, with eight other people, so that the parents are seated next to each other and there is one child on a side of each parent? (Two seatings are considered the same if one can be rotated to look like the other.)