

## Homework 10 CS 275 Discrete Mathematics Fall 2006

### Problem 1

For  $n \in \mathbb{N}$ , consider all paths from  $(0,0)$  to  $(2n,0)$  using the moves  $(x,y) \rightarrow (x+1,y+1)$  and  $(x,y) \rightarrow (x+1,y-1)$ , where no such path can ever fall below the  $x$ -axis. How many such paths are there for each  $n \in \mathbb{N}$ ? (Verify your claim!)

### Problem 2

In each of the following,  $f: \mathbb{Z}^+ \rightarrow \mathbb{R}$ . Solve for  $f(n)$  relative to the given set  $S$ , and determine the appropriate “big-Oh” form for  $f$  on  $S$ .

- a)  $f(1) = 0$        $f(n) = 2f(n/5) + 3$ ,  $n = 5, 25, 125, \dots$   
 $S = \{5^i \mid i \in \mathbb{N}\}$
- b)  $f(1) = 1$        $f(n) = f(n/2) + 2$ ,  $n = 2, 4, 8, \dots$   
 $S = \{2^i \mid i \in \mathbb{N}\}$

### Problem 3

When Cathy and Jill play checkers, each has probability 0.5 of winning. There is never a tie, and the games are independent in the sense that no matter how many games the girls have played, each girl still has probability 0.5 of winning the next game. After each game, the loser gives the winner a quarter. If Cathy has \$2.00 to play with and Jill has \$2.50 and they play until one of them is broke, what is the probability that Cathy gets wiped out?

### Problem 4

Find all (loop-free) nonisomorphic undirected graphs with four vertices. How many of these graphs are connected?

### Problem 5

For  $n \in \mathbb{Z}^+$ , how many distinct (though isomorphic) paths of length 2 are there in the  $n$ -dimensional hypercube  $Q_n$ ?

### Problem 6

Carolyn and Richard attended a party with three other married couples. At this party a good deal of handshaking took place, but (1) no one shook hands with her or his spouse; (2) no one shook hands with herself or himself; (3) no one shook hands with anyone more than once. Before leaving the party, Carolyn asked the other seven people how many hands she or he had shaken. She received a different answer from each of the seven. How many times did Carolyn shake hands at this party? How many times did Richard?

### Problem 7

Are any of the planar graphs for the five Platonic solids bipartite? (Carefully show which ones if any, and show that the other ones are not).