

Homework 11 CS 275 Discrete Mathematics Fall 2006

Problem 1

Give an example of an undirected graph $G = (V, E)$, where $\chi(G) = 3$ but no subgraph of G is isomorphic to K_3 . Prove that your example satisfies the requirements.

Problem 2

For $n \geq 1$, let $a_n = \binom{n}{2}$, the number of edges in K_n , and let $a_0 = 0$. Find the generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$.

Problem 3

On the first Sunday of 2003 Rizzo and Frenchie start a chain letter, each of them sending five letters (to ten different friends between them). Each person receiving the letter is to send five copies to five new people on the Sunday following the letter's arrival. After the first seven Sundays have passed, what is the total number of chain letters that have been mailed? How many were mailed on the last three Sundays?

Problem 4

A code for $\{a, b, c, d, e\}$ is given by $a:00$ $b:01$ $c:101$ $d:x10$ $e:yz1$, where $x, y, z \in \{0, 1\}$. Determine x, y, z so that the given code is a prefix code.

Problem 5

Let $G = (V, E)$ be a loop-free connected undirected graph with biconnected components B_1, B_2, \dots, B_8 . For $1 \leq i \leq 8$ the number of distinct spanning trees for B_i is n_i . How many distinct spanning trees exist for G ?

Problem 6

Let $T = (V, E)$ be a rooted tree with root r . Define the relation \mathfrak{R} on V by $x\mathfrak{R}y$, for $x, y \in V$, if $x = y$ or if x is on the path from r to y . Prove that \mathfrak{R} is a partial order.

Problem 7

- Let $T = (V, E)$ be a complete 6-ary tree of height 8. If T is balanced, but not full, determine the minimum and maximum values for $|V|$.
- Answer part a) if $T = (V, E)$ is a complete m -ary tree of height h .