Homework 2 CS 275 Discrete Mathematics Fall 2006

Problem 1

Construct the truth table for
$(p \to q) \wedge (\neg p \to r)$.

Problem 2

Express the negation of the statement $p \leftrightarrow q$ in terms of the connectives $\wedge, \vee$ and $\neg$. Simplify the expression to disjunctive normal form (a disjunction of conjunctions).

Problem 3

Let p, q, r denote primitive statements. Write the converse, inverse and contrapositive of

a) $p \to (q \wedge r)$

b) $(p \vee q) \to r$

Problem 4

Let p, q, r, s denote primitive statements. Write the dual of each of the following compound statements (T denotes true and F denotes false).

a) $(p \vee \neg q) \wedge (\neg r \vee s)$

b) $p \to (q \wedge \neg r \wedge s)$

c) $[(p \vee T) \wedge (q \vee F)] \vee [r \wedge s \wedge T]$

Problem 5

Establish (through formal derivation) the validity of the argument
$[(p \to q) \wedge [(q \wedge r) \to s] \wedge r] \to (p \to s)$.

Problem 6

Write the following argument in symbolic form. Then either establish the validity of the argument or provide a counterexample to show that it is invalid.
"If it is cool this Friday, then Craig will wear his suede jacket if the pockets are mended. The forecast for Friday calls for cool weather, but the pockets have not been mended. Therefore Craig won't be wearing his suede jacket this Friday."

Problem 7

Determine whether each of the following statements is true or false. Prove your conclusion. The universe comprises all integers.

a) $\forall x \; \exists y \; \exists z \; (x = 7y + 5z)$

b) $\forall x \; \exists y \; \exists z \; (x = 4y + 6z)$