Homework 4 CS 275 Discrete Mathematics Fall 2006

Problem 1

Let a,b,c belong to Z^+ with gcd(a,b)=1. If a|bc, prove that a|c.

Problem 2

For $n \in Z^+$, prove each of the following by mathematical induction:
a) $5 \mid (n^5 - n)$
b) $6 \mid (n^3 + 5n)$

Problem 3

Frances spends \$6.20 on candy for prizes in a contest. If a 10-ounce box of this candy costs \$.50 and a 3-ounce box costs \$.20, how many boxes of each size did she purchase?

Problem 4

Let n be a fixed positive integer that satisfies the property:
For all $a, b \in Z^+$ if n|ab then n|a or n|b. Prove that n=1 or n is prime.

Problem 5

Prove that for all $n \in N$, $H_{2^n} \leq 1 + n$, where H_i denotes the i-th harmonic number.

Problem 6

Let $m, n \in Z^+$ with $19m + 90 + 8n = 1998$. Determine m, n so that
a) n is minimal.
b) m is minimal.

Problem 7

Leslie selects a random integer between 1 and 100 (inclusive). Find the probability her selection is divisible by

(a) 2 or 3.
(b) 2,3 or 5.