Homework 5 CS 275 Discrete Mathematics Fall 2006

Problem 1

Let $A, B \subseteq N$ be subsets of the integers with $1 < |A| < |B|$. If there are 262144 relations from A to B, determine all possibilities for $|A|$ and $|B|$.

Problem 2

Let $\Re \subseteq Z^+ \times Z^+$ be the relation given by the following recursive definition
1) $(1,1) \in \Re$ and
2) For all $(a,b) \in \Re$, the three ordered pairs $(a+1,b),(a+1,b+1)$ and $(a+1,b+2)$ are also in $\Re$.
Prove that $2a \geq b$ for all $(a,b) \in \Re$.

Problem 3

With both of their parents working, Thomas, Stuart, and Graig must handle ten weekly chores among themselves. (a) In how many ways can they divide up the work so that everyone is responsible for at least one chore? (b) In how many ways can the chores be assigned if Thomas, as the eldest, must mow the lawn (one of the weekly chores) an no one is allowed to be idle?

Problem 4

Let A_1, A, B be sets with $\{1,2,3,4,5\} = A_1 \subset A$ and $B = \{s,t,u,v,w,x\}$ and $f : A_1 \to B$. If f can be extended to A in 216 ways, what is $|A|$?

Problem 5

Let S be a set of seven positive integers the maximum of which is at most 24. Prove that the sums of the elements in all the nonempty subsets of S cannot be distinct.

Problem 6

Which of the following operations $f : Z \times Z \to Z$ have an identity?:

a) $f(x,y) = x + y - xy$

b) $f(x,y) = \max\{x,y\}$

c) $f(x,y) = x^y$

d) $f(x,y) = x + y - 3$

Problem 7

Let $A = \{x,a,b,c,d\}$

a) How many closed binary operations f on A satisfy $f(a,b) = c$?
b) How many of the functions f in part (a) have x as an identity?
c) How many of the functions f in part (a) have an identity?
d) How many of the functions f in part (c) are commutative?