

## Homework 6 CS 275 Discrete Mathematics Fall 2006

### Problem 1

Let triangle ABC be equilateral, with  $AB=1$ . Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than  $1/3$ .

### Problem 2

How many times must we roll a single die in order to get the same score

- (a) at least twice?
- (b) at least three times?
- (c) at least  $n$  times for  $n \geq 4$ ?

### Problem 3

For each of the following functions  $f : \mathfrak{R} \rightarrow \mathfrak{R}$  determine whether  $f$  is invertible, and, if so, determine  $f^{-1}$ :

- a)  $f = \{(x, y) \mid 2x + 3y = 7\}$
- b)  $f = \{(x, y) \mid ax + by = c, b \neq 0\}$
- c)  $f = \{(x, y) \mid y = x^3\}$
- d)  $f = \{(x, y) \mid y = x^4 + x\}$

### Problem 4

If  $|A| = |B| = 5$ , how many functions  $f : A \rightarrow B$  are invertible?

### Problem 5

Let  $f, g, h : Z^+ \rightarrow \mathfrak{R}$  where  $f \in O(g)$  and  $g \in O(h)$ . Prove carefully that  $f \in O(h)$ .

### Problem 6

If  $A = \{w, x, y, z\}$  determine the number of relations on  $A$  that are (a) reflexive; (b) symmetric; (c) reflexive and symmetric; (d) reflexive and contain  $(x, y)$ ; (e) symmetric and contain  $(x, y)$ ; (f) antisymmetric; (g) antisymmetric and contain  $(x, y)$ ; (h) symmetric and antisymmetric; and (i) reflexive, symmetric, and antisymmetric.

### Problem 7

If  $R$  is a reflexive relation on a set  $A$ , prove that  $R^2$  is also reflexive on  $A$ .