

## Homework 8 CS 275 Discrete Mathematics Fall 2006

### Problem 1

For  $A = \{1,2,3,4,5\}$  and  $B = \{u, v, w, x, y, z\}$ , determine the number of one-to-one functions  $f : A \rightarrow B$  where  $f(1) \neq v, w$ ;  $f(2) \neq u, w$ ;  $f(3) \neq x$  and  $f(4) \neq v, x, y$ .

### Problem 2

In how many ways can four w's, four x's, four y's and four z's be arranged so that there is no consecutive quadruple of the same letter?

### Problem 3

Determine the generating function for the number of ways to distribute 35 pennies (from an unlimited supply) among five children if (a) there are no restrictions; (b) each child gets at least 1 cent; (c) each child gets at least 2 cents; (d) the oldest child gets at least 10 cents; and (e) the two youngest children must each get at least 10 cents.

### Problem 4

Find the coefficient of  $x^{50}$  in  $(x^7 + x^8 + x^9 + \dots)^6$ .

### Problem 5

How can Mary split up 12 hamburgers and 16 hot dogs among her sons Richard, Peter, Christopher, and James in such a way that James gets at least one hamburger and three hot dogs, and each of his brothers gets at least two hamburgers but at most five hot dogs?

### Problem 6

Find a generating function for the number of ways to partition a positive integer  $n$  into positive-integers summands, where each summand appears an odd number of times or not at all.

### Problem 7

Suppose that  $X$  is a discrete random variable with probability distribution given by

$$\Pr(X = x) = \begin{cases} k \left(\frac{1}{4}\right)^x & x = 0,1,2,3,\dots \\ 0 & \text{otherwise} \end{cases}$$

where  $k$  is a constant. Determine (a) the value of  $k$ ; (b)  $\Pr(X = 3)$ ,  $\Pr(X \leq 3)$ ,  $\Pr(X > 3)$ ,  $\Pr(X \geq 2)$  and (c)  $\Pr(X \geq 4 | X \geq 2)$ ,  $\Pr(X \geq 104 | X \geq 102)$ .