### Homework 1 (100 points)

You must complete all calculations and write-up on your own. Verbatim copying of another student's work is forbidden. You must not consult homework solutions from any source.

1. (15 points) Two balls are placed in a box as follows: A fair coin is tossed and a white ball is placed in the box if a head occurs, otherwise a red ball is placed in the box. The coin is tossed again and a red ball is placed in the box if a tail occurs, otherwise a white ball is placed in the box. Balls are drawn from the box three times in succession (always with replacing the drawn ball back in the box). It is found that on all three occasions a red ball is drawn. What is the probability that both balls in the box are red?

2. (15 points) After your yearly checkup, the doctor has some bad news and good news for you. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e. the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don’t have the disease.) The good news is that this is a rare disease, striking only one in 10,000 people. What is the probability that you actually have the disease?

3. (15 points) Seven friends decide to order pizzas by telephone from Pizza4U based on a flyer pushed through their letterbox. Pizza4U has only 4 kinds of pizza, and each person chooses a pizza independently. Bob phones Pizza4U and places the combined pizza order, simply stating how many pizzas of each kind are required. Unfortunately, the precise order is lost, so the chef makes seven randomly chosen pizzas and then passes them to the delivery person.
   a. How many different combined orders are possible?
   b. What is the probability that the delivery boy has the right order?

4. (10 points) Let \( X, Y, Z \) be random variables. Prove that \( X \perp Y | Z \) (i.e. \( X \) and \( Y \) are conditionally independent given \( Z \)) if and only if the joint probability \( P(X = x, Y = y, Z = z) \) can be expressed in the form of \( a(x, z)b(y, z) \). (Hint: Please make sure you understand what it means by “if-and-only-if” and the requirements for a mathematical proof for such a statement. See, for example, [link](http://math.cos.ucf.edu/~bbrigham/bbrigham_files/ifandonlyif.pdf))

5. (0 points) Download and install the object-oriented BRML library from [link](http://web4.cs.ucl.ac.uk/staff/D.Barber/pmwiki/pmwiki.php?n=Brml.Software). If you are not familiar with object-oriented programming in Matlab, check out the article at [link](http://www.mathworks.com/company/newsletters/articles/introduction-to-object-oriented-programming-in-matlab.html).

The following are programming assignments in Matlab. The goal is to get you familiar with BRMLtoolbox. Please turn in your code listing and a printout of your output.

6. (20 points) Exercise 1.11: Implement the soft XOR gate using BRMLToolbox.

7. (20 points) Exercise 1.13: Implement the two-dice example using BRMLToolbox.